TFEE-1 (Practical)

MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE

Dnipro University of Technology

Department of Electrical Engineering

V.S. KHILOV

Guidelines to independent and practical works on discipline THEORETICAL FUNDAMENTALS OF ELECTRICAL ENGINEERING

For full-time students' majoring in 141 "Electric Power, Electrical Engineering and Electromechanical"

Part 2
THREE–PHASE CIRCUITS, POLYHARMONICAL VOLTAGES AND CURRENTS IN CIRCUIT, TRANSIENT ANALISIS OF A LINEAR CIRCUITS

Dnipro
2021

Рекомендовано до видання навчально-методичним відділом (протокол № від за поданням науково-методичної комісії зі спеціальності 141 – Електроенергетика, електротехніка та електромеханіка (протокол № 21/22-01 від 30.08.2021 р.)

Методичні вказівки до самостійних та практичних занять і контрольні завдання з дисципліни "Теоретичні основи електротехніки" (частина 2, розділи “Трифазні кола”, “Полігармонічні струми й напруги у однофазніх і трифазних колах”, “Перехідні процеси у лінійних електричних колах”) для студентів денної та заочної форм навчання за спеціальностями: 141 Електроенергетика, електротехніка та електромеханіка / В.С. Хілов – Дніпро: Національний технічний університет “Дніпровська політехніка” 2021. - 99 р.

Автор: В. С. Хілов, д-р техн. наук, професор.

Методичні вказівки призначено для виконання самостійної роботи і контрольних завдань та проведення практичних занять з дисципліни "Теоретичні основи електротехніки" (частина 2, розділи “Трифазні кола”, “Полігармонічні струми й напруги у однофазніх і трифазних колах”, “Перехідні процеси у лінійних електричних колах”) студентами денної та заочної форм навчання за спеціальностями: 141 Електроенергетика, електротехніка та електромеханіка.

У кожному розділі подано короткі методичні вказівки, типові завдання з рішенням та необхідними поясненнями, а також вихідні дані для виконання самостійно студентами розрахунково-графічних завдань. Наводяться питання для самостійного контролю залишкових знань.

Друкується в редакції автора.

1. BASIC THEORY OF THE DC ELECTRIC CIRCUIT

Table of Contents PART 2, MODULUS 3, 4

FOREWORD

In presented study guides (modulus 3, 4) as to the parameters calculation the of linear electric circuits include the methods of calculation:

– three-phase schemes at harmonic voltages and currents;

– three-phase schemes at symmetric polyharmonical voltages and currents;

– single-phase schemes at availability polyharmonical voltages and currents;

– as well as of chains in the transient work regimes.

This methodological instructions are the direct continuation of methodological instructions as to the calculation of DC circuits, of single-phase sinusoidal currents and voltages and magnetically coupled circuits (modulus 1, 2).

The calculation basic method of the linear electric circuits parameters at presence high harmonics there is the superposition method which is based on Fourier analysis. Presence polyharmonical voltages and currents in electric circuits brings to the deterioration of electromagnetic energy using that quantitatively is connected with the decrease of the power coefficient and to the appearance of the additional distortion power. The distortion power is absent in circuits if curves of current and voltage completely similar to each other, that always is fulfiled in linear circuits at the presence of the sinusoidal sources of electromagnetic power.

Unstationary or transient processes are calculated by methods there are classic, operational, of state variables or by method on basis of Duhamel integral using. The calculation of transient processes by classical approach is based on differential equations deciding with the laws of commutation and Kirchhoff's laws using. For complex electric circuits the most acceptable there is the calculation of transient processes by operational method, when differential equations in the real variable area are replaced by algebraic equations in the images area. If the graph of forcing action has piecewise-linear complex type, then circuit reaction determine by Duhamel integral using. Calculation by the method of state variable prefer to perform at the differential equations digital integrating.

Before the concrete parameters of electric circuits calculation precede epitomize methodological instructions to which there are necessary follow at the calculation of computative-graphic individual task.

In the every part end are presented typical questions at concrete tasks deciding. For the verification of the material learning degree for every student imperatively is recommended on one's own to decide indicated tasks.

Accuracy and the correctness of fulfiled computative-graphic tasks are verified by the teachers of cycle and after the obviation of errors are assumed to interlocution.

The interlocution according to the results of performed computative-graphic tasks presents there is dialog with teacher and answer toes set questions in the context of in question topic, or answers to test tasks. The results of interlocution are estimated as to five mark scale.

1. THE CALCULATION METHODS OF ELECTRIC THREE-PHASE CIRCUITS WITHIN HARMONIC VOLTAGES AND CURRENTS

1.1. Study guides as to the calculation of three-phase harmonic circuits

1. The symmetrical feed source and symmetrical load.
1.1. Calculation three-phase circuit in symmetrical regime to add up to calculation for single-phase circuit and is fulfiled to similarly of calculation single-phase circuit. Any non-symmetric three-phase circuit can be considered as forked circuit with three sources feeds, for calculation its are applied methods which been used for the calculation of complex electric single-phase circuit. For example, for the case the junction of the generator and load phases by Y-connection without neutral wire at calculation currents and voltages can be applied the method of nodes voltage in phasor form.
1.2. If three-phase symmetrical electric circuit is gathered as scheme of symmetrical Y-connected and whereat of the linear wires impedances excellent from zero, then follows to find the equivalent phase impedances, and then under the Ohm's law to find phase $\underline{I}_P$ (line current I_L) current $\underline{I}_P = \underline{E}_P / \underline{Z}_P$, where $\underline{E}_P$ there is source supply phase voltage; $\underline{Z}_P$ there is phase load impedance. Then under the Ohm's law be found phase voltages on circuit's load. In such schemes linear voltage modulus on load U_L in $\sqrt{3}$ times as much of phase voltage U_P modulus ($U_L = \sqrt{3}\, U_P$), and modulus of line and phase current there are equal $I_L = I_P$.
1.3. If three-phase symmetrical electric circuit is gathered as scheme of symmetrical delta-connected at condition that linear wires impedance excellent from zero, follows to transform the given connection of load impedances into equivalent Y-connection and determine linear currents according to stage 1.2 instructions. The load phase currents at symmetrical delta-connected less than line currents in $\sqrt{3}$ times as much ($I_L = \sqrt{3}\, I_P$), and line voltages on load equals to phase voltages $U_L = U_P$. The load phase voltages look for Ohm's low.
The load phase voltages be find under Ohm's low $U_P = \underline{I}_P \cdot \underline{Z}_P$. If the impedance of linear wires are neglect, then voltage on the phase of the feed source is equal to voltage on the phase load $\underline{E}_P = \underline{U}_P$.
1.4. At symmetrical load active, the reactive and apparent powers of three-phase system irrespective of the method of its connection (Y- or delta) is calculated on one phase and are tripled

$$P = 3 \cdot U_p \cdot I_P \cdot \cos\varphi_P = \sqrt{3} \cdot U_L \cdot I_L \cdot \cos\varphi_P;$$

$Q = 3 \cdot U_p \cdot I_P \cdot \sin \varphi_P = \sqrt{3} \cdot U_L \cdot I_L \cdot \sin \varphi_P;$

$S = 3 \cdot U_p \cdot I_P = \sqrt{3} \cdot U_L \cdot I_L.$

1.5. So long as three-phase circuit there is sinusoidal current circuit that processes research into its perform by the same methods and admittances. For these circuts we may will apply the symbolic method of calculation, topographic voltages diagram and vector currents diagram which make calculations more visual.

2. The symmetrical feed source and non-symmetrical load.

2.1. In non-symmetrical three-phase circuits ($\underline{Z}_A \neq \underline{Z}_B \neq \underline{Z}_C$)in that gathered the source and load phases by Y-connected and with the presence impedance in neutral wire follows to determine the neutral bias voltage (potentials different between the common points of source and load) $\underline{U}_{Nn}$ (voltage between the common points of source and load)

$$\underline{U}_{Nn} = \frac{\underline{E}_A \cdot \underline{Y}_A + \underline{E}_B \cdot \underline{Y}_B + \underline{E}_C \cdot \underline{Y}_C}{\underline{Y}_A + \underline{Y}_B + \underline{Y}_C + \underline{Y}_{Nn}},$$

where $\underline{E}_A, \underline{E}_B, \underline{E}_C$ there are phase voltages (EMF) on supply source; $\underline{Y}_A, \underline{Y}_B, \underline{Y}_C$ there are phase conductivities of three-phase scheme branches; $\underline{Y}_{Nn}$ there is neutral wire conductivity.

Into the conductivities of the branches phase are taken into account the wire line impedances. The load phase currents are determined under the Ohm's law $\underline{I}_A = (\underline{E}_A - \underline{U}_{Nn}) \cdot \underline{Y}_A$; $\underline{I}_B = (\underline{E}_B - \underline{U}_{Nn}) \cdot \underline{Y}_B$; $\underline{I}_C = (\underline{E}_C - \underline{U}_{Nn}) \cdot \underline{Y}_C$.

The current in neutral wire determine under the first Kirchhoff's law $\underline{I}_{Nn} = \underline{I}_A + \underline{I}_B + \underline{I}_C$ или по закону Ома $\underline{I}_{Nn} = \underline{U}_{Nn} \cdot \underline{Y}_{Nn}$.

2.2. If non-symmetric load, gathered by Y-connection and these is connecting to power supply without neutral wire and we known linear voltage of three-phase source, that load phase voltage find as to equations

$$\underline{U}_A = \frac{\underline{U}_{AB} \cdot \underline{Y}_B - \underline{U}_{CA} \cdot \underline{Y}_C}{\underline{Y}_A + \underline{Y}_B + \underline{Y}_C}; \quad \underline{U}_B = \frac{\underline{U}_{BC} \cdot \underline{Y}_C - \underline{U}_{AB} \cdot \underline{Y}_A}{\underline{Y}_A + \underline{Y}_B + \underline{Y}_C};$$

$$\underline{U}_C = \frac{\underline{U}_{CA} \cdot \underline{Y}_A - \underline{U}_{BC} \cdot \underline{Y}_B}{\underline{Y}_A + \underline{Y}_B + \underline{Y}_C},$$

where $\underline{Y}_A + \underline{Y}_B + \underline{Y}_C$ - took into account the conductions of linear wires at presence in it impedance.

Current in the load phases find under the Ohm's law

$\underline{I}_A = \underline{U}_A \cdot \underline{Y}_A; \underline{I}_B = \underline{U}_B \cdot \underline{Y}_B; \underline{I}_C = \underline{U}_C \cdot \underline{Y}_C.$

2.3. At the load connection by delta, in case negligibly small wires impedance, the phase voltages of power supply and loads are equal each to other, the phase currents in the load follows to determine under the Ohm's law, and currents in linear wires follows to determine under to the first Kirchhoff's law

$\underline{I}_A = \underline{I}_{AB} - \underline{I}_{CA}; \underline{I}_B = \underline{I}_{BC} - \underline{I}_{AB}; \underline{I}_C = \underline{I}_{CA} - \underline{I}_{BC}.$

2.4. At the impedances presence in linear wires follows to fulfil the equivalent transform the load connection from triangle into star. After that determine phase voltages in according to stage 2.2 instructions and linear currents under the Ohm's law

$\underline{I}_A = \underline{U}_A \cdot \underline{Y}_A; \underline{I}_B = \underline{U}_B \cdot \underline{Y}_B; \underline{I}_C = \underline{U}_C \cdot \underline{Y}_C.$

The load phase voltages of equivalent Y-connection also are determined under the Ohm's law

$\underline{U}_{AN} = \underline{I}_A \cdot \underline{Z}_{AN}; \underline{U}_{BN} = \underline{I}_B \cdot \underline{Z}_{BN}; \underline{U}_{CN} = \underline{I}_C \cdot \underline{Z}_{CN}.$

According to second Kirchhoff's law can determine the phase drop voltages on the load impedances at connection as triangle

$\underline{U}_{AB} = \underline{U}_A - \underline{U}_B; \underline{U}_{BC} = \underline{U}_B - \underline{U}_C; \underline{U}_{CA} = \underline{U}_C - \underline{U}_A.$

Under the Ohm's law we determine load phase currents

$\underline{I}_{AB} = \underline{U}_{AB} / \underline{Z}_{AB}; \underline{I}_{BC} = \underline{U}_{BC} / \underline{Z}_{BC}; \underline{I}_{CA} = \underline{U}_{CA} / \underline{Z}_{CA}.$

2.5. Active and reactive powers are determined as sum of the active and reactive powers of the load phases, of the line wires and neutral wire

$$P = U_A \cdot I_A \cdot \cos\varphi_A + U_B \cdot I_B \cdot \cos\varphi_B + U_C \cdot I_C \cdot \cos\varphi_C +$$

$$+ U_{Nn} \cdot I_{Nn} \cdot \cos\varphi_{Nn};$$

$$Q = U_A \cdot I_A \cdot \sin\varphi_A + U_B \cdot I_B \cdot \sin\varphi_B + U_C \cdot I_C \cdot \sin\varphi_C +$$

$$+ U_{Nn} \cdot I_{Nn} \cdot \sin\varphi_{Nn}.$$

An apparent power in non-symmetric load

$$S = \sqrt{P^2 + Q^2}.$$

3. The non-symmetrical feed source and symmetrical load.

3.1. Calculation is conducted on basis of the method symmetrical components. In the feed source EMF are separated out symmetrical components of zero, positive and negative phase-sequences.

The zero phase-sequences EMF

$\underline{E}_{A0} = \underline{E}_A + \underline{E}_B + \underline{E}_C; \underline{E}_{B0} = \underline{E}_{C0} = \underline{E}_{A0}.$

The positive phase-sequences EMF

$$\underline{E}_{A1} = \frac{\underline{E}_A + a\underline{E}_B + a^2\underline{E}_C}{3}; \underline{E}_{B1} = a^2\underline{E}_A; \underline{E}_{C1} = a\underline{E}_A,$$

where $a = 1 \cdot e^{j120^0}$ - unit rotary multiplier.

The negative phase-sequences EMF

$$\underline{E}_{A2} = \frac{\underline{E}_A + a^2\underline{E}_B + a\underline{E}_C}{3}; \underline{E}_{B2} = a\underline{E}_A; \underline{E}_{C2} = a^2\underline{E}_A.$$

3.2. At the connection load's impedances in symmetrical star with impedance in neutral wire the phase currents symmetrical components are determined as

$$\underline{I}_{A0} = \frac{\underline{E}_{A0}}{\underline{Z}_P + 3\underline{Z}_{Nn}}; \underline{I}_{B0} = \underline{I}_{C0} = \underline{I}_{A0}.$$

$$\underline{I}_{A1} = \frac{\underline{E}_{A1}}{\underline{Z}_P}; \underline{I}_{B1} = a^2\underline{I}_{A1}; \underline{I}_{C1} = a\underline{I}_{A1},$$

$$\underline{I}_{A2} = \frac{\underline{E}_{A2}}{\underline{Z}_P}; \underline{I}_{B2} = a\underline{I}_{A2}; \underline{I}_{C2} = a^2\underline{E}_{A2}.$$

3.3. At the connection load's impedances in symmetrical star without impedance in neutral wire the phase currents symmetrical components are determined as

$$\underline{I}_{A0} = 0; \underline{I}_{B0} = \underline{I}_{C0} = \underline{I}_{A0}.$$

$$\underline{I}_{A1} = \frac{\underline{E}_{A1}}{\underline{Z}_P}; \underline{I}_{B1} = a^2\underline{I}_{A1}; \underline{I}_{C1} = a\underline{I}_{A1},$$

$$\underline{I}_{A2} = \frac{\underline{E}_{A2}}{\underline{Z}_P}; \underline{I}_{B2} = a\underline{I}_{A2}; \underline{I}_{C2} = a^2\underline{E}_{A2}.$$

3.3. At the connection load's impedances in symmetrical triangle the phase currents symmetrical components are determined as

$$\underline{I}_{AB0} = \frac{\underline{E}_{A0} - \underline{E}_{B0}}{\underline{Z}_P}; \underline{I}_{BC0} = \underline{I}_{CA0} = \underline{I}_{AD0}.$$

$$\underline{I}_{AB1} = \frac{\underline{E}_{A1} - \underline{E}_{B1}}{\underline{Z}_P}; \underline{I}_{BC1} = a^2\underline{I}_{AB1}; \underline{I}_{CA1} = a\underline{I}_{AB1},$$

$$\underline{I}_{AB2} = \frac{\underline{E}_{A2} - \underline{E}_{B2}}{\underline{Z}_P}; \underline{I}_{BC2} = a\underline{I}_{A2}; \underline{I}_{CA2} = a^2\underline{E}_{AB2}.$$

3.4. Active and reactive powers at non-symmetrical feed source and symmetrical load are determined on basis of the method symmetrical components

$$P = 3 \cdot U_{A0} \cdot I_{A0} \cdot \cos\varphi_{A0} + 3 \cdot U_{B0} \cdot I_{B0} \cdot \cos\varphi_{B0} + 3 \cdot U_{C0} \cdot I_{C0} \cdot \cos\varphi_{C0};$$

$$Q = 3 \cdot U_{A0} \cdot I_{A0} \cdot \sin\varphi_{A0} + 3 \cdot U_{B0} \cdot I_{B0} \cdot \sin\varphi_{B0} + 3 \cdot U_{C0} \cdot I_{C0} \cdot \sin\varphi_{C0};$$

An apparent power

$$S = \sqrt{P^2 + Q^2}.$$

Voltages and current of zero phase-sequences having in own phases in every instant the same significance present single-phase current which equally divide between three phases of system. The presence of voltage and current zero phase-sequences even in symmetrical load brings to the appearance the beatings of instant power, i.e. system to become unbalanced.

1.2. The calculation of the circuit parameters at the load gather in symmetrical delta connection

Task. Into three-phase circuit with linear voltage $U_L = 220$V included load, which connected by triangle. The impedance in each phase is $\underline{Z}_\Delta = 10 + 10j$, Ohm (Fig. 1.1). Find currents in each phase of load and line, calculate wattmeter indications. Draw a superpose vector diagrams of currents and voltages.

Task solving.

1. Calculation currents fulfil by symbolic method. We accept the vector of the linear voltage of three-phase the voltage source $\underline{U}_{AB}$ is furnished to real axis, and the impedance of linear wires neglected, that is why can write down

$\underline{U}_{AB} = \underline{U}_{ab} = 220e^{j0^0}$ B; $\underline{U}_{BC} = \underline{U}_{bc} = 220e^{-j120^0}$ B; $\underline{U}_{CA} = \underline{U}_{ca} = 220e^{j120^0}$ B.
We determine load phase currents

$$\underline{I}_{AB} = \underline{U}_{ab} / \underline{Z}_{\Delta} = 220e^{j0^0} /(10+10j) = 11-11j = 15{,}556e^{-j45^0}\,\text{Îì} \quad ;$$

$$\underline{I}_{BC} = \underline{U}_{bc} / \underline{Z}_{\Delta} = 220e^{-j120^0} /(10+10j) = -15{,}026-4{,}026j =$$

$$= 15{,}556e^{-j165^0} = I_{AB}e^{-j120^0}\,\text{Îì} \quad ;$$

$$\underline{I}_{CA} = \underline{U}_{ca} / \underline{Z}_{\Delta} = 220e^{j120^0} /(10+10j) = 4{,}026+15{,}025j =$$

$$= 15{,}556e^{j75^0} = I_{AB}e^{j120^0}\,\text{Îì} \quad .$$

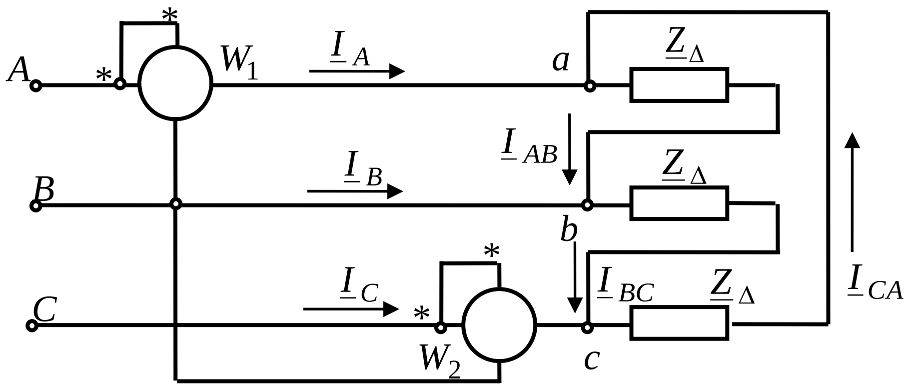


Fig. 1.1

We find linear currents on the grounds of the first Kirchhoff's law

$$\underline{I}_A = \underline{I}_{AB} - \underline{I}_{CA} = 6{,}96 - 25{,}98j = 26{,}9e^{-j75} A;$$

$$\underline{I}_B = \underline{I}_{BC} - \underline{I}_{AB} = -25{,}98 - 6{,}96j = 26{,}9e^{-j195^0} = \underline{I}_A e^{-j120} A;$$

$$\underline{I}_C = \underline{I}_{CA} - \underline{I}_{BC} = 19{,}02 + 19{,}02j = 26{,}9e^{j45} = \underline{I}_A e^{j120} A.$$

We determine wattmeter indications

$$P_1 = \mathrm{Re}\left[\underline{U}_{AB} \cdot \overset{*}{\underline{I}}_A\right] = \mathrm{Re}[220e^{j0^0} \cdot 26{,}9e^{j45^0}] = 220 \cdot 26{,}9 \cdot \cos 45^0 = 1530\text{Âò};$$

$$P_2 = \mathrm{Re}\left[\underline{U}_{CB} \cdot \overset{*}{\underline{I}}_C\right] = \mathrm{Re}[-220e^{-j120^0} \cdot 26{,}9e^{-j45^0}] =$$

$$= \mathrm{Re}[220e^{j60^0} \cdot 26{,}9e^{-j45^0}] = 220 \cdot 26{,}9 \cdot \cos 15^0 = 5730\text{Âò};$$

The circuit active power is determined the as algebraic sum of wattmeter indications, i.e.

$$P = P_1 + P_2 = 1530 + 5730 = 7260\text{Âò}$$

or

$$P = \sqrt{3} \cdot U_L \cdot I_L \cdot \cos\varphi_P = \sqrt{3} \cdot 220 \cdot 26{,}9 \cdot \cos 45^0 = 7260 Wt.$$

The superpose vector diagrams of currents and voltages is given on Fig.1.2.

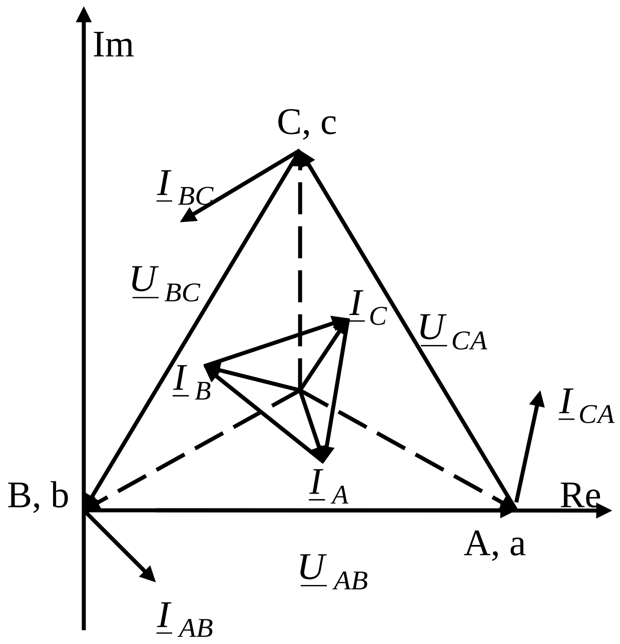


Fig. 1.2

1.3. The circuit parameters calculation at the load gather in non-symmetrical Y-connection

Task.

Calculate the parameters of electric circuit for the case of non-symmetric load. In four-wire three-phase scheme with linear voltage $U_L = 220$ V the load impedances connected by star, the resistive and inductive phases resistance accordingly are equal

$R_A = 3\Omega; x_A = 4\Omega;\ R_B = 3\Omega; x_B = 5{,}2\Omega;\ R_C = 4\Omega; x_C = 3\Omega;$ (Fig.1.3).

Determine currents in linear and neutral wires and draw vector diagram.

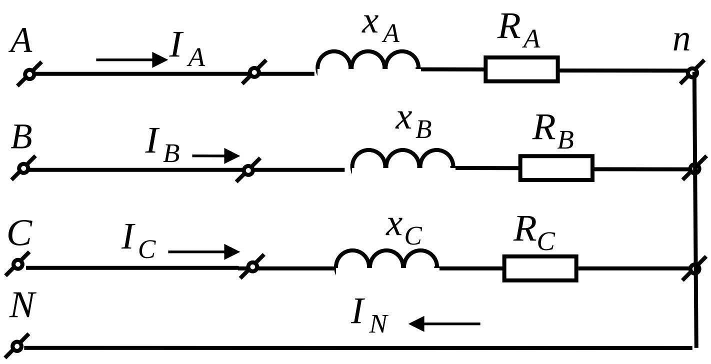


Fig. 1.3

Task solving.

We accept that vector of phase voltage $\underline{U}_A$ is furnished to real axis, then

$\underline{U}_A = U_L / \sqrt{3} e^{j0^0} = 127 e^{j0^0}$ B; $\underline{U}_B = 1270 e^{-j120^0}$ B; $\underline{U}_C = \underline{U}_{ca} = 127 e^{j120^0}$ B.

We find the linear currents:

$\underline{I}_A = \underline{U}_A / \underline{Z}_A = 127/(4+4j) = 25{,}4e^{-j53^0}\ A;$

$\underline{I}_B = \underline{U}_B / \underline{Z}_B = 127e^{-j120^0}\ /(3+5{,}2j) = 21{,}2e^{-j180^0}\ A;$

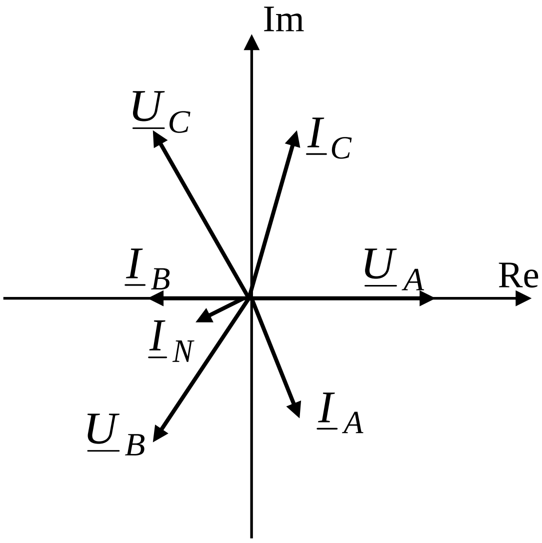


Рис. 1.4

$\underline{I}_C = \underline{U}_C / \underline{Z}_C = 127e^{j120^0}\ /(4+3j) = 25{,}4e^{j83^0}\ A.$

The current in neutral wire determine as sum of the phasor phases current

$\underline{I}_N = \underline{I}_A + \underline{I}_B + \underline{I}_C =$

$= 25{,}4e^{-j53^0} + 21{,}2e^{-j180^0} + 25{,}4e^{j83^0} = 5{,}9e^{j124^0}\ A.$

At non-symmetricly load the active power find as sum of phase powers

$P = U_A \cdot I_A \cdot \cos\varphi_A + U_B \cdot I_B \cdot \cos\varphi_B +$

$+ U_C \cdot I_C \cdot \cos\varphi_C =$

$= 127 \cdot 25{,}4 \cdot \cos 53^0 + 127 \cdot 21{,}2 \cdot \cos 60^0 +$

$+ 127 \cdot 25{,}4 \cdot \cos 37^0 = 5863{,}77 Wt.$

The superpose vector diagrams of currents and voltages is given on Fig.1.4.

1.4. The circuit parameters calculation at the load gather in non-symmetrical Y-connection as to three-wire and four-wire schemes

Task. Three-wire circuit.

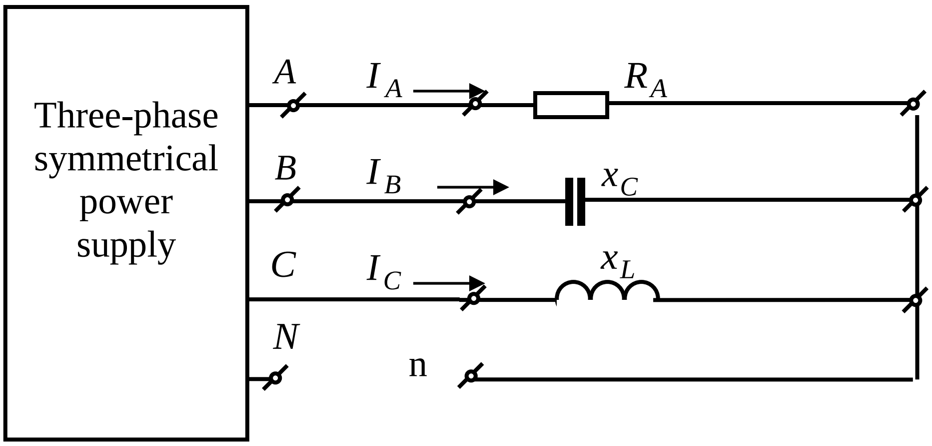


Fig. 1.5

In three-wire and three-phase scheme with linear voltage $U_L = 380$ V the load impedances connected by star, the resistive, capacitive and inductive phases resistance accordingly are equal: $R = x_L = x_C = 22\Omega$ (Fig.1.5). The neutral wire is absent. Determine currents in the load phases. Draw superpose vector diagram currents and voltages.

Task solving.

Having chosen phase A as initial, we distribute the phase voltages of the symmetrical feed source on plane of complex numbers

$\underline{U}_A = U_L / \sqrt{3} e^{j0^0} = 220 e^{j0^0}$,V; $\underline{U}_B = 220 e^{-j120^0} = -110 - 191j$,V;

$\underline{U}_C = 220 e^{j120^0} = -110 + 191j$,V.

We determine voltage between the neutral points of the feed source and load (the neutral bias voltage)

$$\underline{U}_{Nn} = \frac{\underline{U}_A \cdot \underline{Y}_A + \underline{U}_B \cdot \underline{Y}_B + \underline{U}_C \cdot \underline{Y}_C}{\underline{Y}_A + \underline{Y}_B + \underline{Y}_C} = \frac{\frac{220}{22} + \frac{-110-191j}{-j22} + \frac{-110+191j}{j22}}{\frac{1}{22} + \frac{1}{-j22} + \frac{1}{j22}} = 602 \text{, V}.$$

We determine the voltage on the clamps of the load phases

$\underline{U}_{An} = \underline{U}_A - \underline{U}_{Nn} = 220 - 602 = -382 = 382 e^{j180^0}$ B;

$\underline{U}_{Bn} = \underline{U}_B - \underline{U}_{Nn} = -110 - 191j - 602 = -712 - 191j = 737{,}17 e^{-j165^0}$ B;

$\underline{U}_{Cn} = \underline{U}_C - \underline{U}_{Nn} = -110 + 191j - 602 = -712 + 191j = 737{,}17 e^{j165^0}$ B.

We determine phase (linear) currents

$$\underline{I}_A = (\underline{U}_A - \underline{U}_{Nn}) \cdot \underline{Y}_A = \frac{-382}{22} = -17{,}3 = 17{,}3 e^{j180^0} \text{ A;}$$

$$\underline{I}_B = (\underline{U}_B - \underline{U}_{Nn}) \cdot \underline{Y}_B = \frac{-712 - 191j}{-22j} = 8{,}68 - 32{,}4j = 33{,}54 e^{-j75^0} \text{ A;}$$

$$\underline{I}_C = (\underline{U}_C - \underline{U}_{Nn}) \cdot \underline{Y}_C = \frac{-712 + 191j}{22j} = 8{,}68 + 32{,}4j = 33{,}54 e^{j75^0} \text{ A.}$$

At non-symmetrical load the active power find as sum of phase powers

$P = U_{An} \cdot I_A \cdot \cos\varphi_A +$
$+ U_{Bn} \cdot I_B \cdot \cos\varphi_B +$
$+ U_{Cn} \cdot I_C \cdot \cos\varphi_C =$
$= 220 \cdot 17{,}3 \cdot \cos 180^0 +$
$+ 220 \cdot 33{,}54 \cdot \cos 195^0 +$
$+ 220 \cdot 33{,}54 \cdot \cos 195^0 =$
$= 18060{,}74 Wt.$

The superpose vector diagrams of currents and voltages is given on Fig.1.6.

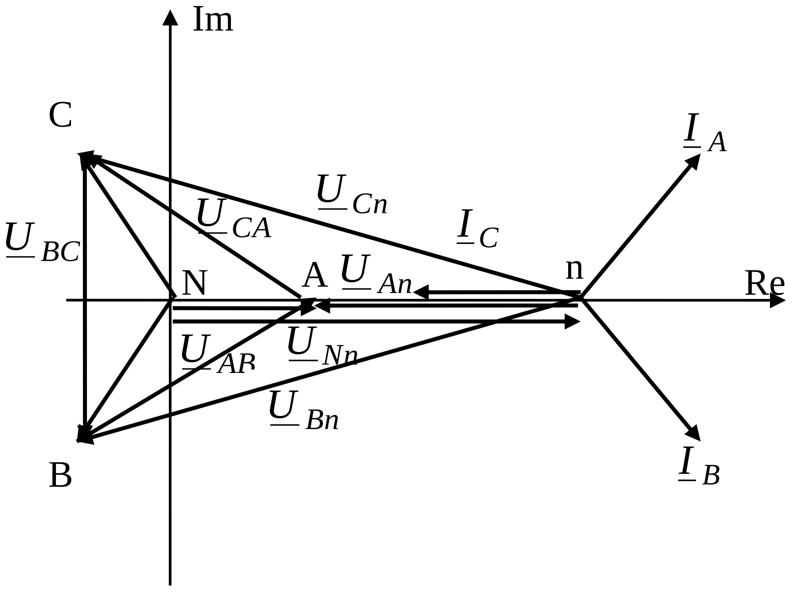


Fig. 1.6

Task.
Four-wire circuit.
For the conditions of previous task (Fig. 1.5 three-wire circuit $U_L = 380V$, $R = x_L = x_C = 22\Omega$), but at closing by the neutral wire points N and n in scheme (Fig.1.7 four-wire circuit), currents in the phases of load to define.

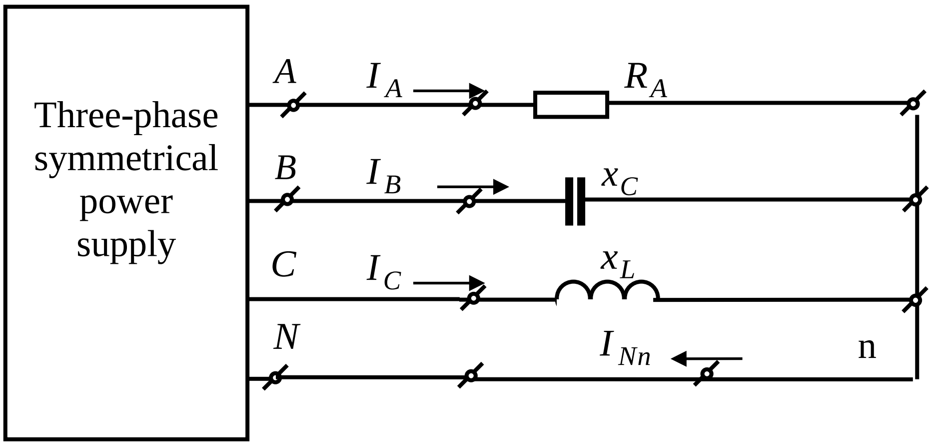


Fig. 1.7

Task solving.
We preserve accepted distribution of phase and linear voltages of the three-phase symmetric power supply. We preserve phase A as initial, distribute of phase voltages of the symmetrical feed source on plane of complex numbers $\underline{U}_A = U_L / \sqrt{3} e^{j0^0} = 220 e^{j0^0}$ B; $\underline{U}_B = 220 e^{-j120^0} = -110 - 191j$ B;
$\underline{U}_C = 220 e^{j120^0} = -110 + 191j$ B.
We determine voltage between the neutral points of the feed source and load (the neutral bias voltage)

$$\underline{U}_{Nn} = \frac{\underline{U}_A \cdot \underline{Y}_A + \underline{U}_B \cdot \underline{Y}_B + \underline{U}_C \cdot \underline{Y}_C}{\underline{Y}_A + \underline{Y}_B + \underline{Y}_C + \underline{Y}_{Nn}} = \frac{\dfrac{220}{22} + \dfrac{-110-191j}{-j22} + \dfrac{-110+191j}{j22}}{\dfrac{1}{22} + \dfrac{1}{-j22} + \dfrac{1}{j22} + \infty} = 0, \text{ V.}$$

Just like was awaited the voltage of bias neutral is equal to zero, i.e. on complex plane the potentials of points N and n coincide, they are equipotentials.
Voltages on the phases of load are equal to voltages on the phases of the feed source

$$\underline{U}_{An} = \underline{U}_A - \underline{U}_{Nn} = 220 - 0 = 220 = 220 e^{j0^0}, \text{ V};$$

$$\underline{U}_{Bn} = \underline{U}_B - \underline{U}_{Nn} = -110 - 191j - 0 = -110 - 191j = 220 e^{-j120^0}, \text{ V};$$

$$\underline{U}_{Cn} = \underline{U}_C - \underline{U}_{Nn} = -110 + 191j - 0 = -110 + 191j = 220 e^{j120^0}, \text{ V}.$$

We determine phase (linear) currents

$$\underline{I}_A = (\underline{U}_A - \underline{U}_{Nn}) \cdot \underline{Y}_A = \frac{220}{22} = 11 = 11e^{j0^0} \text{ A};$$

$$\underline{I}_B = (\underline{U}_B - \underline{U}_{Nn}) \cdot \underline{Y}_B = \frac{-110-191j}{-22j} = 8{,}68 - 5j = 10e^{-j30^0} \text{ A};$$

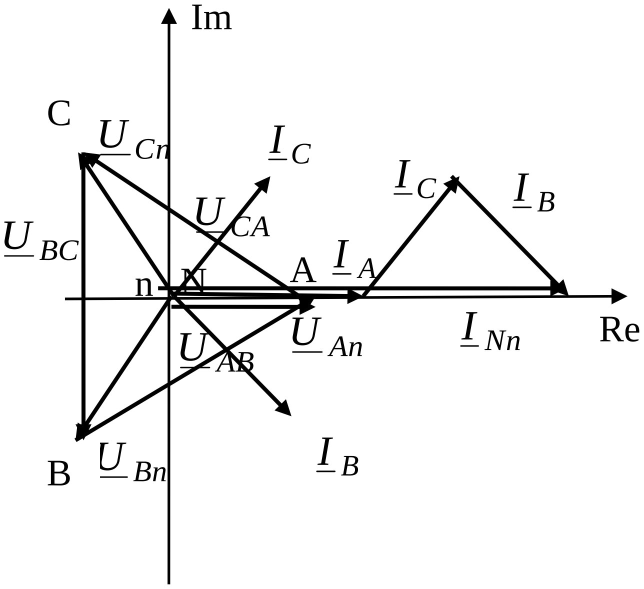


Fig. 1.8

$$\underline{I}_C = (\underline{U}_C - \underline{U}_{Nn}) \cdot \underline{Y}_C = \frac{-110+191j}{22j} =$$

$$= 8{,}68 + 5j = 10e^{j30^0}, A.$$

Current in neutral wire we find as sum of the phases significances of phases current

$$\underline{I}_{Nn} = \underline{I}_A + \underline{I}_B + \underline{I}_C = 11 + 8{,}68 + 5j + \text{A.}$$

$$+8{,}68 - 5j = 28{,}36 = 28{,}36e^{j0^0}, A.$$

Active power is allocated only in the phase A ohmic resistance

$$P = R_A I_A^2 = 22 \cdot 11^2 = 2662 Wt.$$

The superpose vector diagrams of currents and voltages is given on Fig.1.8.

1.5. The circuit parameters calculation at the load gather in non-symmetrical connection and when there are impedances in lines

Will calculate parameters of three-phase circuit at presence line impedances.

Task.

Non-symmetric three-phase circuit comprises symmetrical power supply, Fig.1.9. Calculation is simplified, if reduce the calculation to determining of the

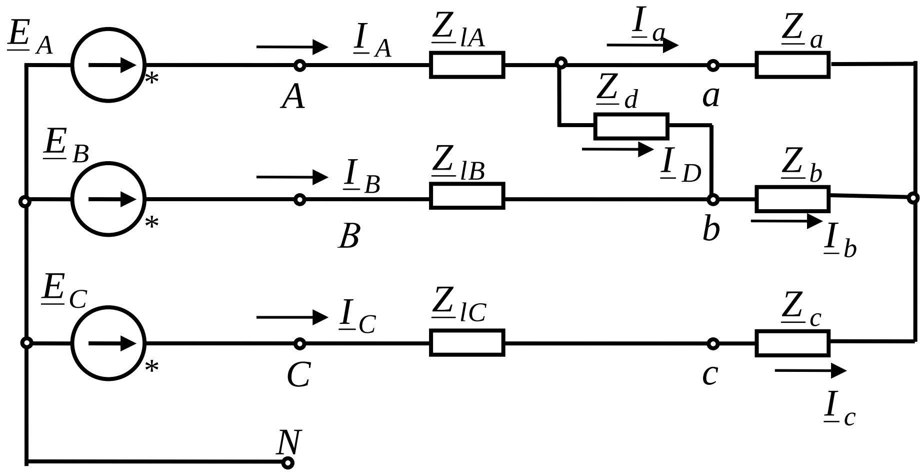


Fig. 1.9

scheme parameters in that impedances gathered in star. At the load and power supply connections as star, by the most convenient method of calculation there is method with determining of the voltage of bias neutral which is found on the method of node potentials.

Initial conditions for the calculation of parameters non-symmetric three-phase circuit:

E=380, V; $\underline{Z}_{lA}$= $\underline{Z}_{lB}$=$\underline{Z}_{lC}$=(3+4j), Ohm; $\underline{Z}_a$= (15-8j), Ohm; $\underline{Z}_b$= (15-8j), Ohm; $\underline{Z}_c$= (10+12j), Ohm; $\underline{Z}_d$ = (10+14), Ohm.

Task solving.

We accept the initial phase of EMF the phase A as initial. In the calculation symbolic form distribute the phase EMF of the symmetrical feed source on complex plane

$\underline{E}_A = 380e^{j0^0}$, V; $\underline{E}_B = 380e^{-j120^0}$, V; $\underline{E}_C = 380e^{j120^0}$, V.

We shall transform the star impedances $\underline{Z}_a$, $\underline{Z}_b$, $\underline{Z}_c$ into equivalent triangle $\underline{Z}_{ab}$, $\underline{Z}_{bc}$, $\underline{Z}_{ca}$ (Fig. 10)

$$\underline{Z}_{ab} = \underline{Z}_a + \underline{Z}_b + \frac{\underline{Z}_a \cdot \underline{Z}_b}{\underline{Z}_c} =$$

$$= 15 - 8j + 15 - 8j + \frac{(15-8j)\cdot(15-8j)}{10+12j} = (24{,}795 - 33{,}754) = 41{,}882e^{-j53^0}\ Oì\ ;$$

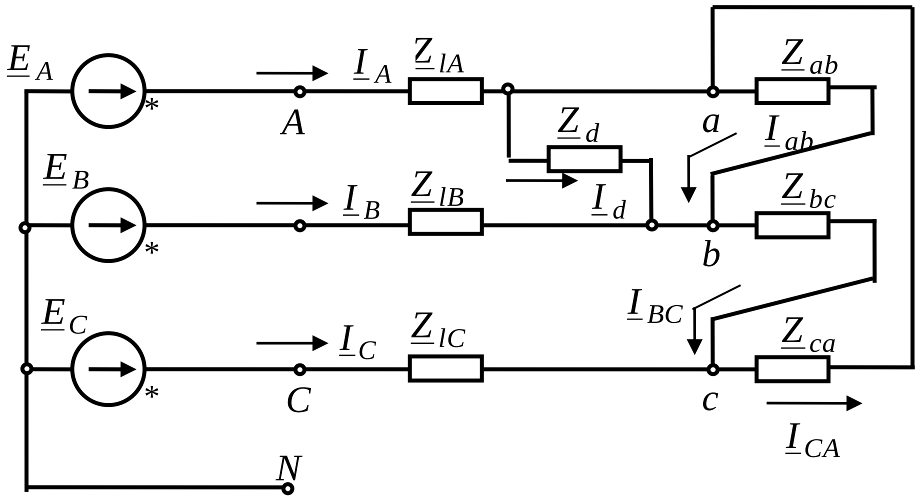


Fig. 1.10

Equivalent delta-connected impedances we calculate as to following known equations

$$\underline{Z}_{bc} = \underline{Z}_b + \underline{Z}_c + \frac{\underline{Z}_b \cdot \underline{Z}_c}{\underline{Z}_a} =$$

$$= 15 - 8j + 10 + 12j + \frac{(15-8j)\cdot(10+12j)}{15-8j} = (35{,}0 + 16{,}0j) = 38{,}483e^{j24^0}\ \Omega;$$

$$\underline{Z}_{ca} = \underline{Z}_c + \underline{Z}_a + \frac{\underline{Z}_c \cdot \underline{Z}_a}{\underline{Z}_b} =$$

$$= 10 + 12j + 15 - 8j + \frac{(15-8j)\cdot(10+12j)}{15-8j} = (35{,}0 + 16{,}0j) = 38{,}483e^{j24^0}\,\Omega;$$

We reduce to one equivalent impedance of resistance in the triangle phase AB

$$\underline{Z}_{abd} = \frac{\underline{Z}_d \cdot \underline{Z}_{ab}}{\underline{Z}_d + \underline{Z}_{ab}} = \frac{(10+14j)\cdot(24{,}795-33{,}754j)}{(10+14j)+(24{,}795-33{,}754j)} =$$

$$= (15{,}541 + 9{,}098) = 18{,}0e^{j30^0}\,\Omega.$$

We find the branch impedances and conductions of the of the three-phase equivalent star with allowance for impedances in lines (Fig. 1.11)

$$\underline{Z}_A = \frac{\underline{Z}_{abd} \cdot \underline{Z}_{ca}}{\underline{Z}_{abd} + \underline{Z}_{bc} + \underline{Z}_{ca}} + \underline{Z}_{la} = \frac{(15{,}541+9{,}098j)\cdot(35+16j)}{(15{,}541+9{,}098j)+(35+16j)+(35+16j)} +$$

$$+ (3+4j) = (9{,}371 + 7{,}560j) = 12{,}041e^{-j38^0}\,\Omega;$$

$$\underline{Z}_B = \frac{\underline{Z}_{bc} \cdot \underline{Z}_{abd}}{\underline{Z}_{abd} + \underline{Z}_{bc} + \underline{Z}_{ca}} + \underline{Z}_{la} = \frac{(15{,}541+9{,}098j)\cdot(35+16j)}{(15{,}541+9{,}098j)+(35+16j)+(35+16j)} +$$

$$+ (3+4j) = (9{,}371 + 7{,}560j) = 12{,}041e^{-j38^0}\,\Omega;$$

$$\underline{Z}_C = \frac{\underline{Z}_{bc} \cdot \underline{Z}_{ca}}{\underline{Z}_{abd} + \underline{Z}_{bc} + \underline{Z}_{ca}} + \underline{Z}_{la} = \frac{(35+16j)\cdot(35+16j)}{(15{,}541+9{,}098j)+(35+16j)+(35+16j)} +$$

$$+ (3+4j) = (17{,}387 + 10{,}212) = 20{,}164e^{-j30^0}\,\Omega.$$

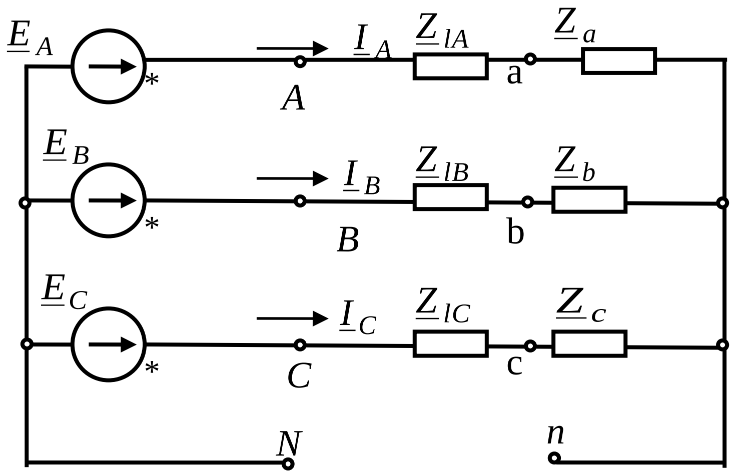


Fig. 1.11

$$\underline{Y}_A = \frac{1}{\underline{Z}_A} = \underline{Y}_B = \frac{1}{\underline{Z}_B} = \frac{1}{(9{,}731 + 7{,}56j)} = 0{,}064 - 0{,}0497j = 0{,}081e^{-j37^0}\,S;$$

$$\underline{Y}_C = \frac{1}{\underline{Z}_C} = \frac{1}{(17{,}387 + 10{,}212j)} = 0{,}0427 - 0{,}0251j = 0{,}0495e^{-j30^0}\,S;$$

We find the bias neutral voltage and linear (phase) currents in every equivalent scheme phase. Calculation we perform on the method of two nodes (the method of node potentials the particular case) basis

$$\underline{U}_{Nn} = \frac{\underline{E}_A \cdot \underline{Y}_A + \underline{E}_B \cdot \underline{Y}_B + \underline{E}_C \cdot \underline{Y}_C}{\underline{Y}_A + \underline{Y}_B + \underline{Y}_C} =$$

$$= \frac{380e^{j0^0} \cdot 0{,}081e^{-j37^0} + 380e^{-j120^0} \cdot 0{,}081e^{-j37^0} + 380e^{j120^0} \cdot 0{,}0495e^{-j30^0}}{0{,}081e^{-j37^0} + 0{,}081e^{-j37^0} + 0{,}0495e^{-j30^0}} =$$

$$= 16{,}774 - 58{,}649j = 60{,}84e^{-j74}, V.$$

Under the Ohm's law we reckon phase currents (Fig. 1.11) and find fitting conjugate values

$$\underline{I}_A = (\underline{E}_A - \underline{U}_{Nn}) \cdot \underline{Y}_A = (380e^{j0^0} - 60{,}84e^{-j74^0}) \cdot 0{,}081e^{-j37^0} =$$

$$= 26{,}517 - 15{,}157j = 30{,}543e^{-j30^0} A;$$

$$\overset{*}{\underline{I}}_A = 26{,}517 + 15{,}157j = 30{,}543e^{j30^0} A;$$

$$\underline{I}_B = (\underline{E}_B - \underline{U}_{Nn}) \cdot \underline{Y}_B = (380e^{-j120^0} - 60{,}84e^{-j74^0}) \cdot 0{,}081e^{-j37^0} =$$

$$= -27{,}467 - 6{,}681j = 28{,}261e^{-j166^0} A;$$

$$\overset{*}{\underline{I}}_B = -27{,}467 + 6{,}681j = 28{,}261e^{j166^0} A;$$

$$\underline{I}_C = (\underline{E}_C - \underline{U}_{Nn}) \cdot \underline{Y}_V = (380e^{j120^0} - 60{,}84e^{-j74^0}) \cdot 0{,}0495e^{-j30^0} =$$

$$= 0{,}943 + 21{,}838j = 21{,}859e^{j87^0} A;$$

$$\overset{*}{\underline{I}}_C = 0{,}943 - 21{,}838j = 21{,}859e^{-j87^0} A;$$

We determine the potentials values on clamps phase impedances and voltage drops in wires feeding lines (Fig. 1.11)

$$\underline{V}_a = \underline{E}_A - \underline{Z}_{lA} \cdot \underline{I}_A = 380e^{j0^0} - (3 + 4j) \cdot 30{,}543e^{-j30^0} =$$

$$= 239{,}82 - 60{,}597j = 247{,}357e^{-j14^0}, V;$$

$$\underline{V}_b = \underline{E}_B - \underline{Z}_{lB} \cdot \underline{I}_B = 380e^{-j120^0} - (3 + 4j) \cdot 28{,}261e^{-j166^0} =$$

$$= -134{,}345 - 199{,}209j = 240{,}276e^{-j124^0}, V;$$

$$\underline{V}_c = \underline{E}_C - \underline{Z}_{lC} \cdot \underline{I}_C = 380e^{j120^0} - (3 + 4j) \cdot 21{,}859e^{-j87^0} =$$

$$= -105{,}475 - 259{,}805j = 280{,}4e^{-j112^0}, V;$$

$$\Delta\underline{U}_{lA} = \underline{E}_A - \underline{V}_a = 380e^{j0^0} - 247{,}357e^{-j14^0} =$$

$$= 140{,}181 + 60{,}597j = 152{,}717e^{j23^0}, V;$$

$$\Delta\underline{U}_{lB} = \underline{E}_B - \underline{V}_b = 380e^{-j120^0} - 240{,}276e^{-j124^0} =$$

$$= -55{,}66 - 129{,}88j = 141{,}3e^{-j113^0}, V;$$

$$\Delta\underline{U}_{lC} = \underline{E}_C - \underline{V}_c = 380e^{j120^0} - 280{,}4e^{-j112^0} =$$
$$= -84{,}525 + 69{,}284j = 109{,}293e^{j140^0}, V.$$
We find the phase voltages in every load phase of the equivalent scheme
$$\underline{U}_{an} = \underline{V}_a - \underline{U}_{Nn} = 247{,}357e^{-j14^0} - 60{,}84e^{-j74} = 223{,}045 + 1{,}947j =$$
$$= 223{,}054e^{-j0{,}5^0}, V;$$
$$\underline{U}_{bn} = \underline{V}_b - \underline{U}_{Nn} = 240{,}276e^{-j124^0} - 60{,}84e^{-j74} = -151{,}119 - 140{,}56j =$$
$$= 206{,}383e^{-j137^0}, V;$$
$$\underline{U}_{cn} = \underline{V}_c - \underline{U}_{Nn} = 280{,}4e^{-j112^0} - 60{,}84e^{-j74} = -122{,}249 + 318{,}455j =$$
$$= 341{,}113e^{j111^0}, V.$$
We reckon currents through impedances in initial scheme Fig.1.10
$$\underline{I}_d = \frac{\underline{U}_{an} - \underline{U}_{bn}}{\underline{Z}_d} = \frac{223{,}054e^{-j0{,}5^0} - 206{,}383e^{-j137^0}}{10 + 14j} = 19{,}197 - 13{,}014j =$$
$$= 23{,}2e^{-j34^0} A;$$
$$\underline{I}_a = \underline{I}_A - \underline{I}_d = 30{,}543e^{-j30^0} - 23{,}2e^{-j34^0} = 7{,}32 - 2{,}14j = 7{,}628e^{-j16^0} A;$$
$$\underline{I}_b = \underline{I}_B + \underline{I}_d = 28{,}261e^{-j166^0} + 23{,}2e^{-j34^0} = -8{,}263 - 19{,}695j = 21{,}36e^{-j112^0} A;$$
$$\underline{I}_c = \underline{I}_C = 0{,}943 + 21{,}838j = 21{,}859e^{j87^0} A;$$
We determine the potential of the load's neutral point in initial scheme Fig. 1.9.
$$\underline{V}_n = \underline{E}_C - (\underline{Z}_{lC} + \underline{Z}_c) \cdot \underline{I}_C = 380e^{j120^0} - (3 + 4j10 + 12j) \cdot 21{,}859e^{-j87^0} =$$
$$= 147{,}159 + 30{,}114j = 150{,}4e^{j11^0}, V.$$
We calculate the active powers of symmetrical three-phase power supply and non-symmetric load
$$P_s = \mathrm{Re}\left[\underline{E}_A \cdot \overset{*}{\underline{I}}_A + \underline{E}_{\hat{A}} \cdot \overset{*}{\underline{I}}_{\hat{A}} + \underline{E}_{\tilde{N}} \cdot \overset{*}{\underline{I}}_{\tilde{N}}\right] = \mathrm{Re}[380e^{j0^0} \cdot 30{,}543e^{j30^0} +$$
$$+ 380e^{-j120^0} \cdot 28{,}2613e^{j166^0} + 380e^{j120^0} \cdot 21{,}859e^{-j87^0}] = 2{,}45 \cdot 10^3, Wt;$$
$$P_l = R_l \cdot (I_A^2 + I_B^2 + I_C^2) + R_d \cdot I_d^2 + R_a \cdot I_a^2 + R_b \cdot I_b^2 + R_c \cdot I_c^2 =$$
$$= 3 \cdot \left(30{,}543^2 + 28{,}261^2 + 21{,}859^2\right) + 10 \cdot 23{,}2^2 +$$
$$+ 15 \cdot 7{,}628^2 + 10 \cdot 21{,}859^2 = 2{,}45 \cdot 10^3, Wt.$$

Relative error for engineering calculations should not exceed 5% and for carried out calculations composes value
$$\gamma = (|P_s - P_l|)/P_l \cdot 100\% = 0\%$$
We draw on scale currents vector diagram and topographic voltages diagram, Fig.1.12. Topographic diagram illustrates voltages distribution between various points of the three-phase circuit. Drawing begins from the choice of convenient voltages and currents scale and dispositions of the power supply N neutral point

on complex plane. As a rule, this point situate of at the coordinates origin, i.e. the potential the N point of the feed source is accepted to equal zero. Relatively this point are postponed the phase and the linear voltages of the symmetrical feed source. In drawing take into account, that linear voltages are determined through fitting of phase voltages. Subtracting from the points potentials vales of the built equilateral the triangle voltage apices of the symmetrical feed source the voltage drops on line impedances, we shall receive the point potentials which determine the potentials of the apices of linear voltages scalene triangle on load. From the onset of coordinates (point N) postpone the bias neutral vector between neutral points, i.e. find the potential of point n on load which is connected as star. Having connected point n with the apices of linear voltages scalene triangle we shall receive phase voltages vectors in the elements connection initial scheme. The current vectors suitably and visually to postpone from the neutral point of n load.

Decompose the line currents system into symmetrical components of zero, positive and negative phase-sequences. Zero phase-sequences is absent, because in connection by star without neutral wire is fulfiled condition $\underline{I}_A+\underline{I}_B+\underline{I}_C=0$, i.e.

$$\underline{I}_{A0}=\frac{\underline{I}_A+\underline{I}_B+\underline{I}_C}{3}=\frac{30{,}543e^{-j30^0}+28{,}261e^{-j166^0}+21{,}859e^{j87^0}}{3}=0;.$$

$$\underline{I}_{B0}=\underline{I}_{C0}=0.$$

We distinguish positive currents phase-sequence

$$\underline{I}_{A1}=\frac{\underline{I}_A+a\underline{I}_B+a^2\underline{I}_C}{3}=$$

$$=\frac{30{,}543e^{-j30^0}+1\cdot e^{j120^0}\cdot 28{,}261e^{-j166^0}+1\cdot e^{j240^0}\cdot 21{,}859e^{j87^0}}{3}=$$

$$=21{,}491-15{,}778j=26{,}661e^{-j36^0}A;$$

$$\underline{I}_{B1}=a^2\underline{I}_{A1}=1\cdot e^{j240^0}\cdot 26{,}661e^{-j36^0}=26{,}661e^{j204^0}A;$$

$$\underline{I}_{C1}=a\underline{I}_{A1}=1\cdot e^{j120^0}\cdot 26{,}661e^{-j36^0}=26{,}661e^{j84^0}A.$$

The negative currents phase-sequence

$$\underline{I}_{A2}=\frac{\underline{I}_A+a^2\underline{I}_B+a\underline{I}_C}{3}=$$

$$=\frac{30{,}543e^{-j30^0}+1\cdot e^{j240^0}\cdot 28{,}261e^{-j166^0}+1\cdot e^{j120^0}\cdot 21{,}859e^{j87^0}}{3}=$$

$$=5{,}026+0{,}62j=5{,}064e^{j7^0}A;$$

$$\underline{I}_{B2}=a\underline{I}_{A1}=1\cdot e^{j120^0}\cdot 5{,}064e^{j7^0}=5{,}064e^{j127^0}A;$$

$$\underline{I}_{C2}=a^2\underline{I}_{A1}=1\cdot e^{j240^0}\cdot 5{,}064e^{j7^0}=5{,}064e^{j247^0}A.$$

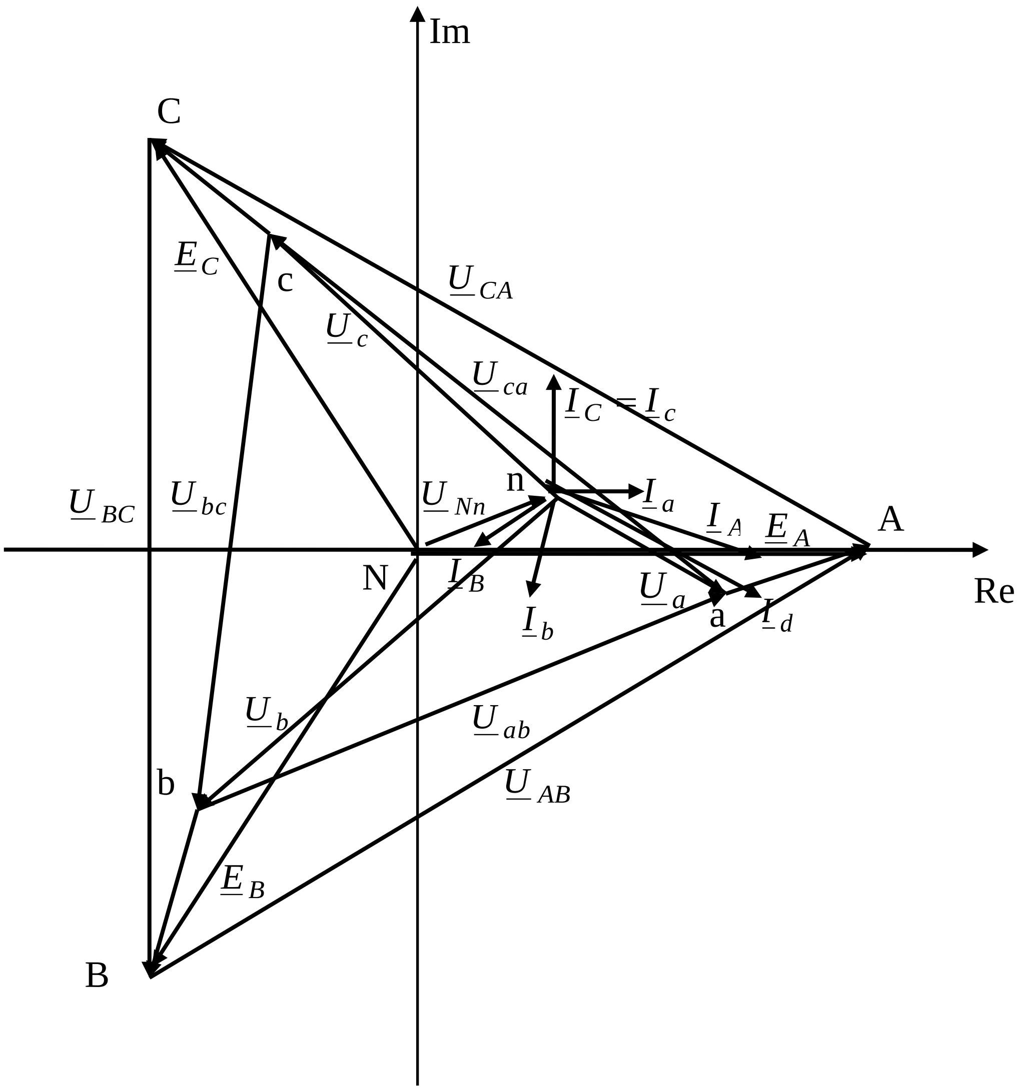


Fig. 1.12

Graphic currents decomposition on positive (Fig. 1.13) and negative (Fig. 1.14) phase-sequences consists in drawing of current vectors and their sums according to known analytic expressions.

On complex plane postpone current vector $\underline{I}_A$, by the end of this vector add vector $\underline{I}_B$, which turn on corner 120^0, to the last vector add vector $\underline{I}_C$, which turn on corner 240^0. We connect the onset of the first vector with end of last and received vector divide by three equal parts. Found one third there is current vector of the phase A positive phase-sequence. The current phase B vector of the positsve phase-sequence advance on 240^0 phase A current vector of the positive phase-sequence, and in phase C lags behind on 120^0 phase A current vector of the positive phase-sequence,

Similarly is built the current vectors system for negative phase-sequence.

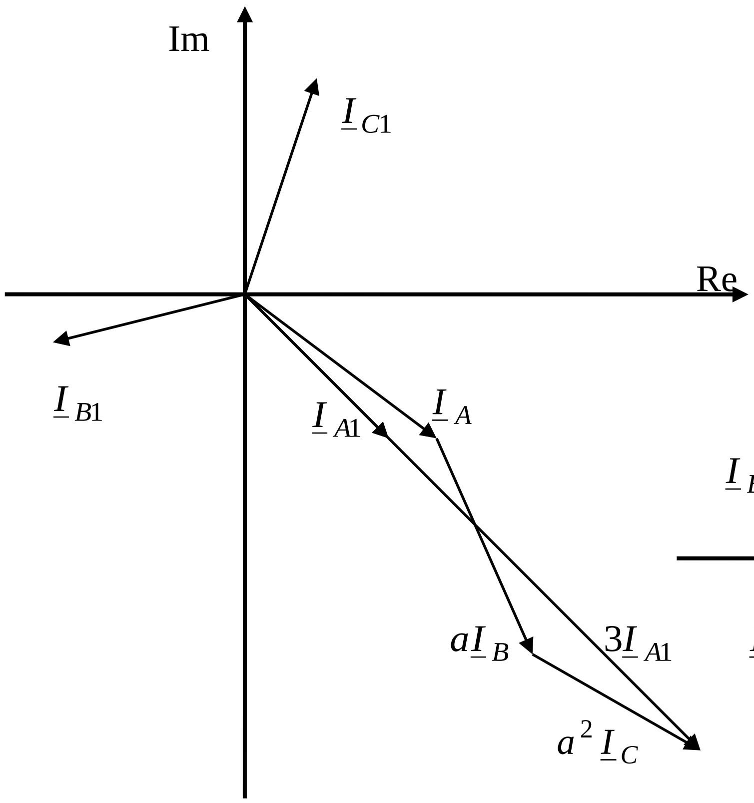


Fig. 1.13

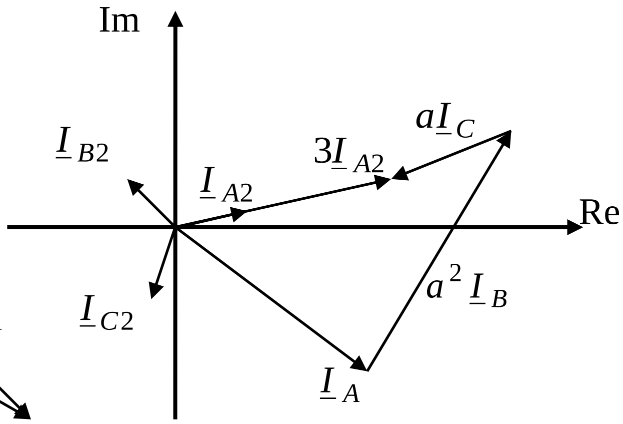


Fig. 1.14

1.6. The personal computative-graphic task "The calculation parameters of three-phase circuit at harmonic voltages and currents"

Calculation non-symmetrical three-phase circuit.
On Fig.1.15 is given non-symmetric three-phase scheme with symmetrical phase electromotive forces (EMF). Numeral significances of EMF, of line phasor impedances and load are given in Table 1.1. The three-phase inner impedances neglect.

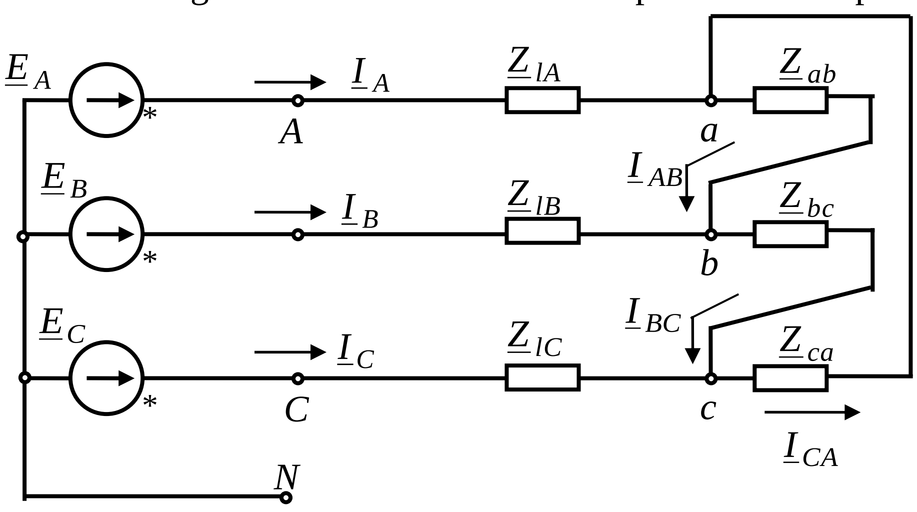


Fig. 1.15

For given circuit Fig1.15 it is necessary do following calculations:

1. Determine currents and voltages on all circuit districts.
2. Compose power balances.
3. Build in scale currents vector diagram and the topographic diagram of potentials.
4. Decompose received linear currents system on symmetrical phase-sequences by analytically and graphically.

Table 1.1.

Variant	E, V	$\underline{Z}_{lA}$,Ω	$\underline{Z}_{lB}$,Ω	$\underline{Z}_{lC}$,Ω	$\underline{Z}_{ab}$,Ω	$\underline{Z}_{bc}$,Ω	$\underline{Z}_{ca}$,Ω
01	660	1	0	0	10+10j	10+15j	10-10j
02	380	0	1	0	10-10j	10+10j	10+15j
03	220	0	0	1	10+15j	10-10j	10+10j
04	127	1j	0	0	10+5j	10+15j	10-10j
05	660	0	1j	0	10-10j	10+5j	10+15j
06	380	0	0	1j	10+15j	10-10j	10+5j
07	220	-1j	0	0	20+10j	10+15j	10-10j
08	127	0	-1j	0	10-10j	20+10j	10+15j
09	660	0	0	-1j	10+15j	10-10j	20+10j
10	380	2	0	0	20+5j	10+15j	10-10j
11	220	0	2	0	10-10j	20+5j	10+15j
12	127	0	0	2	10+15j	10-10j	20+5j
13	660	2j	0	0	10-10j	10+15j	10-10j
14	380	0	2j	0	10-10j	10-10j	10+15j
15	220	0	0	2j	10+15j	10-10j	10-10j
16	127	-2j	0	0	10-5j	10+15j	10-10j
17	660	0	-2j	0	10-10j	10-5j	10+15j
18	380	0	0	-2j	10+15j	10-10j	10-5j
19	220	1+2j	0	0	20-10j	10+15j	10-10j
20	127	0	1+2j	0	10-10j	20-10j	10+15j
21	660	0	0	1+2j	10+15j	10-10j	20-10j
22	380	1+1j	0	0	20-5j	10+15j	10-10j
23	220	0	1+1j	0	10-10j	20-5j	10+15j
24	127	0	0	1+1j	10+15j	10-10j	20-5j
25	660	1-1j	0	0	10+10j	10+15j	10-10j
26	380	0	1-1j	0	10-10j	10+10j	10+15j
27	220	0	0	1-1j	10+15j	10-10j	10+10j
28	127	2+1j	0	0	10+5j	10+15j	10-10j
29	660	0	2+1j	0	10-10j	10+5j	10+15j
30	380	0	0	2+1j	10+15j	10-10j	10+5j
31	220	2+2j	0	0	20+10j	10+15j	10-10j

Continuation of Table 1.1.

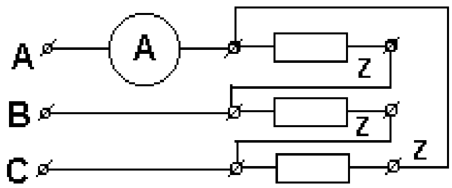
Variant	E, V	$\underline{Z}_{lA}$,Ω	$\underline{Z}_{lB}$,Ω	$\underline{Z}_{lC}$,Ω	$\underline{Z}_{ab}$,Ω	$\underline{Z}_{bc}$,Ω	$\underline{Z}_{ca}$,Ω
32	127	0	2+2j	0	10-10j	20+10j	10+15j
33	660	0	0	2+2j	10+15j	10-10j	20+10j
34	380	1-2j	0	0	20+5j	10+15j	10-10j
35	220	0	1-2j	0	10-10j	20+5j	10+15j
36	127	0	0	1-2j	10+15j	10-10j	20+5j
37	660	2-2j	0	0	10-10j	10+15j	10-10j
38	380	0	2-2j	0	10-10j	10-10j	10+15j
39	220	0	0	2-2j	10+15j	10-10j	10-10j
40	127	0	1,5	0	10-5j	10+15j	10-10j
41	660	1	0	0	10+15j	10-10j	20-10j
42	380	0	1	0	20-5j	10+15j	10-10j
43	220	0	0	1	10-10j	20-5j	10+15j
44	127	1j	0	0	10+15j	10-10j	20-5j
45	660	0	1j	0	10+10j	10+15j	10-10j
46	380	0	0	1j	10-10j	10+10j	10+15j
47	220	-1j	0	0	10+15j	10-10j	10+10j
48	127	0	-1j	0	10+5j	10+15j	10-10j
49	660	0	0	-1j	10-10j	10+5j	10+15j
50	380	2	0	0	10+15j	10-10j	10+5j
51	220	0	2	0	20+10j	10+15j	10-10j
52	127	0	0	2	10-10j	20+10j	10+15j
53	660	2j	0	0	10+15j	10-10j	20+10j
54	380	0	2j	0	20+5j	10+15j	10-10j
55	220	0	0	2j	10-10j	20+5j	10+15j
56	127	-2j	0	0	10+15j	10-10j	20+5j
57	660	0	-2j	0	10-10j	10+15j	10-10j
58	380	0	0	-2j	10-10j	10-10j	10+15j
59	220	1+2j	0	0	10+15j	10-10j	10-10j
60	127	0	1+2j	0	10-5j	10+15j	10-10j
61	660	0	0	1+2j	10+10j	10+15j	10-10j
62	380	1+1j	0	0	10-10j	10+10j	10+15j
63	220	0	1+1j	0	10+15j	10-10j	10+10j
64	127	0	0	1+1j	10+5j	10+15j	10-10j
65	660	1-1j	0	0	10-10j	10+5j	10+15j
66	380	0	1-1j	0	10+15j	10-10j	10+5j
67	220	0	0	1-1j	20+10j	10+15j	10-10j
68	127	2+1j	0	0	10-10j	20+10j	10+15j
69	660	0	2+1j	0	10+15j	10-10j	20+10j
70	380	0	0	2+1j	20+5j	10+15j	10-10j

Continuation of Table 1.1.

Variant	E, V	$\underline{Z}_{lA}$,Ω	$\underline{Z}_{lB}$,Ω	$\underline{Z}_{lC}$,Ω	$\underline{Z}_{ab}$,Ω	$\underline{Z}_{bc}$,Ω	$\underline{Z}_{ca}$,Ω
71	220	2+2j	0	0	10-10j	20+5j	10+15j
72	127	0	2+2j	0	10+15j	10-10j	20+5j
73	660	0	0	2+2j	10-10j	10+15j	10-10j
74	380	1-2j	0	0	10-10j	10-10j	10+15j
75	220	0	1-2j	0	10+15j	10-10j	10-10j
76	127	0	0	1-2j	10-5j	10+15j	10-10j
77	660	2-2j	0	0	10-10j	10-5j	10+15j
78	380	0	2-2j	0	10+15j	10-10j	10-5j
79	220	0	0	2-2j	20-10j	10+15j	10-10j
80	127	0	1,5	0	10-10j	20-10j	10+15j
81	660	1	0	0	20+10j	10+15j	10-10j
82	380	0	1	0	10-10j	20+10j	10+15j
83	220	0	0	1	10+15j	10-10j	20+10j
84	127	1j	0	0	20+5j	10+15j	10-10j
85	660	0	1j	0	10-10j	20+5j	10+15j
86	380	0	0	1j	10+15j	10-10j	20+5j
87	220	-1j	0	0	10-10j	10+15j	10-10j
88	127	0	-1j	0	10-10j	10-10j	10+15j
89	660	0	0	-1j	10+15j	10-10j	10-10j
90	380	2	0	0	10-5j	10+15j	10-10j
91	220	0	2	0	10+10j	10+15j	10-10j
92	127	0	0	2	10-10j	10+10j	10+15j
93	660	2j	0	0	10+15j	10-10j	10+10j
94	380	0	2j	0	10+5j	10+15j	10-10j
95	220	0	0	2j	10-10j	10+5j	10+15j
96	127	-2j	0	0	10+15j	10-10j	10+5j
97	660	0	-2j	0	20+10j	10+15j	10-10j
98	380	0	0	-2j	10-10j	20+10j	10+15j
99	220	1+2j	0	0	10+15j	10-10j	20+10j
100	127	0	1+2j	0	20+5j	10+15j	10-10j

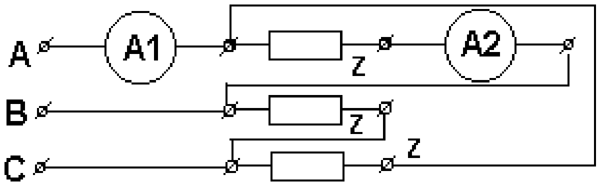
1.7. Questions for one's own checking as to the calculation methods of three-phase harmonic circuits

1. Symmetrical three-phase load there is connected into triangle, and included into three-phase network with voltage U_L=220 V. Find linear current at the load phase resistance R_P=11 Ohm.

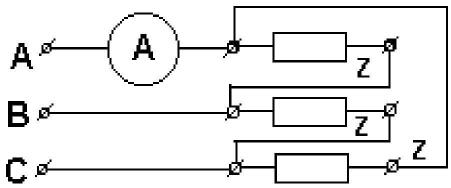


2. Symmetrical three-phase load there is connected in star, and included into three-phase network with voltage U_L=220 V. find linear current at the load phase resistance R_P=11 Ohm.

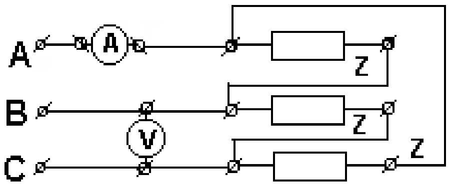
3. Ammeter A1 included in the circuit of symmetrical three-phase load, indication current value 34.6 A. What value current will show ammeter A2?



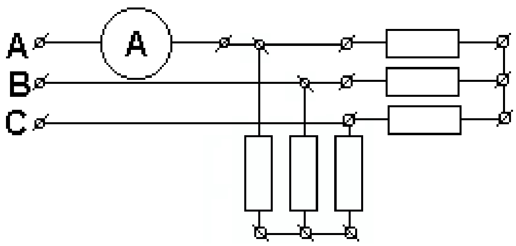
4. System of sinusoidal linear voltages is symmetrical. Find the indications of ammeter, if the known circuit parameters U_L=127 V, $R_P = Z$=10 Ohm.



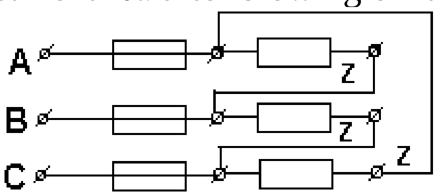
5. The phase resistance of symmetrical three-phase load $R_P = Z$=10 Ohm. What will voltmeter indication if ammeter indicated 17.3 A?



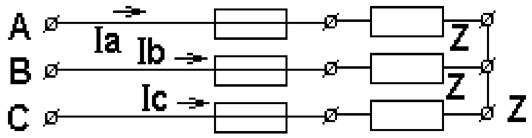
6. In the circuit the linear voltages are sinusoidal and U_L=380 V. All resistance (6 ones) are similar and equal to $R_P = Z$=20 Ohm each. Find ammeter A indication.



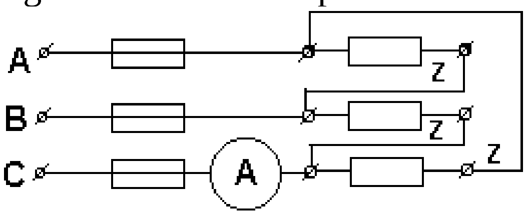
7. Phase currents of symmetrical three-phase load are equal to 15 A each. What will become current Ica after blowing of fuse in wire of phase A?



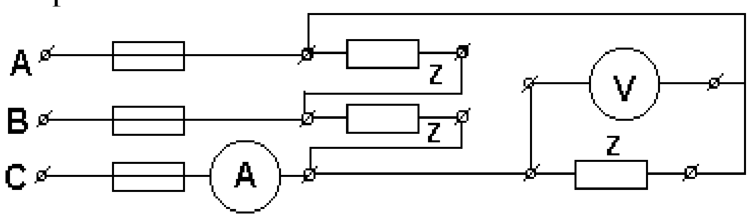
8. Into how many times will change value of active power, if symmetrical load, gathered by star without neutral wire, reconnect into triangle at unchanged linear voltage?
9. The phases resistance of the of couple symmetrical three-phase loads are equivalents. The first load is connected into triangle, second one into star, while of both loads are connected to common network. Find the relation of linear current of the first load to linear current of second load.
10. Symmetrical three-phase load which gathered by triangle, has only ohmic resistance in phase $R_1 = Z_\Delta$ =15 Ohm. Second symmetrical load is gathered by star and connected into same three-phase network. What ohmic phase resistance of second load $R_2 = Z_Y$, if we known, what of linear currents of both wirelesses are equal?
11. Symmetrical three-phase load which gathered by star, has only ohmic resistance in phase $R_1 = Z_Y$ =9 Ohm. Second symmetrical load is gathered by triangle and connected into same three-phase network. What ohmic phase resistance of second load $R_2 = Z_\Delta$, if we known, what of linear currents of both wirelesses are equal?
12. Symmetrical three-phase load there is connected into triangle, and included into three-phase network with voltage U_L =127 V. Find linear current at the load phase resistance R_P=15 Ohm when wire break in A line.
13. Given linear voltage U_P =127 V of three-phase network and the ohmic resistance of 15 Ohm symmetrical three-phase load. Find current in wire A after blowing of fuse in wire of phase C.



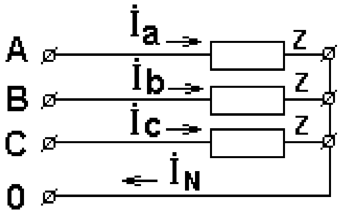
14. Phase currents of symmetrical three-phase load equals to 12 A. What will be current in line C after blowing of fuse in wire of phase A?



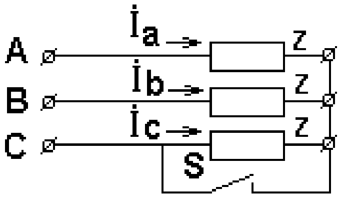
15. Three-phase network feeding symmetrical load has linear voltage U_L =127 V. What will voltmeter indicated which included into phase CA after blowing of fuse in wire of phase C?



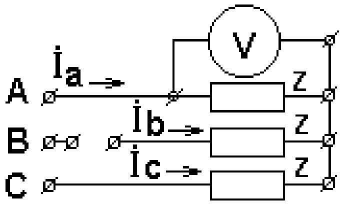
16. Into how many times will change value of line current, if symmetrical load, gathered by star with neutral wire, reconnect into triangle at unchanged linear voltage?
17. How change phase current in symmetrical load, by gathered as star with neutral wire when wire break in A line.? Load is connected to the symmetrical system of voltage source.



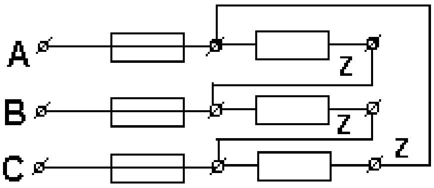
18. Given linear voltage U_L=380 V of three-phase network which is connected to symmetrical three-phase load. What will be voltage in phase B, if in phase C impedance there is short circuit.



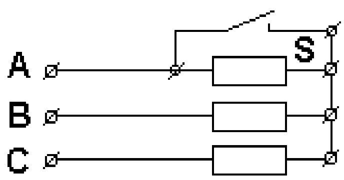
19. What will voltmeter indicate which included in the scheme of symmetrical three-phase load, if linear voltage of feeding networks is equal U_L=220 V, and linear wire break in phase B?



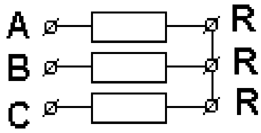
20. Phase currents of symmetrical three-phase load are equal 18 A. What will become current I_{bc} after blowing of fuse in wire B?



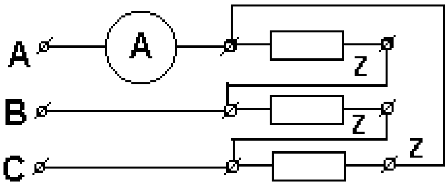
21. Three-phase circuit worked in symmetrical regime. Load is connected by star without null wire. After wire break in phase A necessary to determine modulus of voltages U_B and U_C.
22. How to change linear currents I_B and I_C of symmetrical star without neutral wire, if in phase A load is short circuit? Linear current in symmetrical load before closing switch S was equal to value I=5 A.



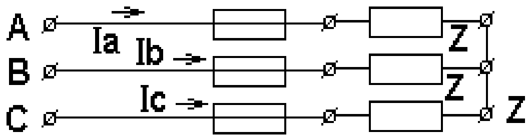
23. Three-phase circuit worked at symmetrical regime. Load is connected by star without null wire. Find voltage modulus in phases B and C (U_B and U_C) after short circuit in phase A.
24. Symmetrical three-phase load there is connected into triangle, and included into three-phase network with voltage U_L=220 V. Find linear current at the load phase resistance R_P=11 Ohm after loss of phase B.



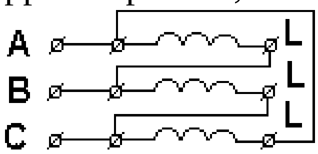
25. Into how many times will change value of active power, if symmetrical load, gathered by triangle, reconnect into star at unchanged linear voltage?
26. Symmetrical three-phase load there is connected into triangle, and included into three-phase network with voltage U_L=220 V. Find consumed active, reactive and apparent powers at the load phase resistance $\underline{Z}_P = 10 + j10$ Ohm.



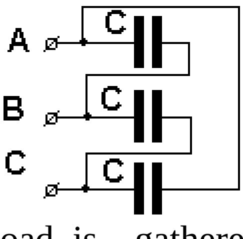
27. Symmetrical three-phase load there is connected into star, and included into three-phase network with voltage U_L=220 V. Find consumed active, reactive and apparent powers at the load phase impedance $\underline{Z}_P = 10 - j10$ Ohm.



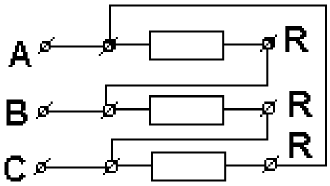
28. Symmetrical three-phase load is gathered from ideal inductance elements as triangle which connected to three-phase circuit by voltage U_L=220 V. Find consumed active, reactive and apparent powers, if current in line B is 5 A.



29. Symmetrical three-phase load is gathered from ideal capacitance elements as triangle which connected to three-phase circuit by voltage U_L=380 V. Find consumed active, reactive and apparent powers, if current in line C is 2 A.



30. Symmetrical three-phase load is gathered from ideal omhic resistance elements as triangle which connected to three-phase circuit by voltage U_L=220 V. Find consumed active, reactive and apparent powers, if current in line B is 5 A.



2. THE CALCULATION METHODS OF ELECTRIC SINGLE-PHASE AND THREE-PHASE CIRCUITS WITHIN NON-HARMONIC VOLTAGES AND CURRENTS

2.1. Study guides as to the calculation of single-phase non-harmonic circuits

1. Non-harmonic currents $i(\omega t)$ and voltages $u(\omega t)$, or in generally case function $f(\omega t)$, are periodical carvers form of which are not sinusoidal. Periodical function $f(\omega t) = f(\omega t + 2\pi)$ satisfying to Dirichlet conditions, maybe presented in the form of the sum endless trigonometrically (harmonically) of Fourier series

$$f(\omega t) = A_0 + \sum_{k=1}^{\infty} A_{km} \sin(k\omega t + \psi_k),$$

where A_0 is constant component; κ is harmonic number (order); A_{km} is κ-th harmonic amplitude; ψ_k is κ-th harmonic initial phase.
In this case non-sinusoidal periodical function is considered as a result of the superposition of sinusoids with aliquot frequencies: $\omega_k = k\omega$, where $\omega = 2\pi / T$ is main (or first) harmonic frequency.
Each harmonic has its own initial phase and amplitude.
Trigonometrically row can be writed down through sinusoidal and co-sinusoidal components, each of which has null initial phase

$$f(\omega t) = A_0 + \sum_{k=1}^{\infty} B_{km} \sin(k\omega t) + \sum_{k=1}^{\infty} C_{km} \cos(k\omega t),$$

and $A^2{}_{km} = B^2{}_{km} + C^2{}_{km}$ $tg\psi_k = C_{km} / B_{km}$ at that.
In its turn can be determined that $B_{km} = A_{km} \cos\psi_k$ и $C_{km} = A_{km} \sin\psi_k$.
Coefficients A_0, B_{km}, C_{km} are determined through initial function $fi(\omega t)$ by means of integrals Fourier

$$A_0 = \frac{1}{2\pi} \int_0^{2\pi} f(\omega t) d\omega t; B_{km} = \frac{1}{2\pi} \int_0^{2\pi} f(\omega t) \cdot \sin k\omega t \cdot d\omega t;$$

$$C_{km} = \frac{1}{2\pi} \int_0^{2\pi} f(\omega t) \cdot \cos k\omega t \cdot d\omega t.$$

2. Periodic non-sinusoidal function can be characterized by discrete frequency the spectra of harmonic amplitudes $A_{km}(\omega)$ and initial phases $\psi_k(\omega)$ which accordingly are called amplitude-frequency and phase-frequency.

Spectral composition determines the form of complex harmonic vibration. Two non-sinusoidal vibrations have similar form only in similar amplitude-frequency and phase-frequency spectra.

3. Determining of spectral composition no-sinusoidal periodical current or voltage can be essential simplified, if preliminary to establish the disposition of the non-sinusoidal periodical curve symmetry relatively coordinate axes.

If the non-harmonic periodical curve symmetrical of coordinates onset, then such curve is odd $f(\omega t) = -f(-\omega t)$ and Fourier series does not comprise null and cosines composing

$$f(\omega t) = \sum_{k=1}^{\infty} B_{km} \sin(k\omega t).$$

If the non-harmonic periodical curve symmetrical of ordinate axe, then such curve is even $f(\omega t) = f(-\omega t)$ and Fourier series does not comprise sines composing

$$f(\omega t) = A_0 + \sum_{k=1}^{\infty} C_{km} \cos(k\omega t).$$

If the non-harmonic periodical curve symmetrical of time axis (abscissa), then for such curve it's true $f(\omega t) = -f(-\omega t + \pi)$ and Fourier series does not comprise null and even composing

$$f(\omega t) = \sum_{k=1,3,5...}^{\infty} A_{km} \sin(k\omega t + \psi_k) = \sum_{k=1,3,5...}^{\infty} B_{km} \sin(k\omega t) + \sum_{k=1,3,5...}^{\infty} C_{km} \sin(k\omega t).$$

There are possible cases, when investigated curve owns the several types of symmetry. If curve symmetrical relatively coordinates onset and abscissas axis, then Fourier series is simplified till form

$$f(\omega t) = \sum_{k=1,3,5...}^{\infty} B_{km} \sin(k\omega t).$$

At the symmetry relatively of y-axises and abscissas, Fourier series changes till form

$$f(\omega t) = \sum_{k=1,3,5...}^{\infty} C_{km} \sin(k\omega t).$$

4. If non-sinusoidal function is described analytical, then determining of its spectral composition is performed by means of the search of amplitudes and initial phases harmonic by means of Fourier formulae.

If there is absent the analytic description of investigated periodical non-sinusoidal function, then in this instance the parameters of harmonic Fourier series possibly to calculate by means of graphic-analytical method. The graphic-analytical method is founded on the replacement of definite integral by the sum of finite number of summands. The constant component and amplitudes sin and cosune components of series detect out from correlations

$$A_0 = \frac{1}{n} \sum_{p=1}^{n} f_p(x); B_{km} = \frac{2}{n} \sum_{p=1}^{n} f_p(x) \sin_p(kx); \quad C_{km} = \frac{2}{n} \sum_{p=1}^{n} f_p(x) \cos_p(kx),$$

where x – running coordinate; n – partitioning number on the repetition period ; p – current index accepting significances from 1 till n; $f_p(x)$ - the significance of harmonic function at current phase $x = p \cdot 2\pi / n$; $\sin_p(kx)$, $\cos_p(kx)$ - the significance of function $\sin(kx)$ and $\cos(kx)$ at current co-ordinate $x = p \cdot 2\pi / n$.

In calculations follows to take into account that omhic resistance accords the identical resistance for all voltage harmonic components $R_k = constant)$. Inductive resistance enlarges with the growth of harmonic number $x_{Lk} = k\omega L$, capacitive resistance decreases with growth of harmonic number $x_{Ck} = 1/(k\omega C)$.

5. For the every calculation be performed by the known methods of circuits single-phase harmonic currents calculation, by namely symbolic method. For every harmonic component taken separately, possibly draw current vector diagram and combined with it vector voltage diagram.

6. At presence higher harmonics in voltage and current curves and heterogeneous reactive elements in circuits are possible resonance phenomena on separate harmonics. If voltage and current on k-th harmonic coincides as to phase, then on this harmonic is seen the voltage resonance in the series connection of heterogeneous reactive elements, in their parallel connection is seen the currents resonance.

7. Voltage (or current) effective value at presence higher harmonics is determined as root square from the sum of squares null component and effective values of harmonic components

$$U = \sqrt{U_0^2 + \sum_{k=1}^{\infty} U_k^2}; \; I = \sqrt{I_0^2 + \sum_{k=1}^{\infty} I_k^2}.$$

8. Coefficients characterizing form of non-sinusoidal periodic curves.

For the characteristic of the periodical curves form are introduced the coefficients of amplitude K_A, forms K_F, distortions K_D, of harmonics K_H and pulsations K_P.

The amplitude coefficient K_A is determined as quotient of maximal value to it effective value

$$K_A = \frac{A_m}{\sqrt{A_0^2 + \sum_{k=1}^{\infty} A_k^2}}.$$

For the harmonic function the amplitude coefficient $K_A = \sqrt{2} = 1{,}41$.

The form coefficient K_F there is the quotient of effective value to middle A_{Md} for the half of period ones

$$K_F = \frac{A}{\grave{A}_{Md}}.$$

For the harmonic function the form coefficient $K_F = 1{,}11$.

The distortion coefficient K_D is determined as quotient of effective value main harmonic component to it effective value

$$K_D = \frac{A_1}{\sqrt{A_0^2 + \sum\limits_{k=1}^{\infty} A_k^2}}.$$

For harmonic function the distortion coefficient $K_D = 1{,}0$.

The harmonic coefficient obtain as quotient of non-sinusoidal effective value not counting zero and the first harmonic to acting value of the first harmonic

$$K_H = \frac{\sqrt{\sum\limits_{k=2}^{\infty} A_k^2}}{\grave{A}_1}.$$

For sinusoidal value the harmonic coefficient is not determined.

The pulsations coefficient is determined as quotient of non-sinusoidal effective value not counting zero and the first harmonic to effective value of the first harmonic

$$K_P = \frac{\sqrt{\sum\limits_{k=2}^{\infty} A_k^2}}{\grave{A}_1}.$$

For sinusoidal value the harmonic coefficient is not determined.

9. The circuit power at alternating current of arbitrary form is determined as middle power for period or in symmetrical curve form for the period half.

Active power in non-harmonic currents and voltages is equal to the sum of powers of separate harmonics

$$P = U_0 I_0 + \sum_{k=1}^{\infty} U_k I_k \cos\varphi_k .$$

Reactive power in non-harmonic currents and voltages is equal to the sum of powers of separate harmonics

$$Q = \sum_{k=1}^{\infty} U_k I_k \sin\varphi_k .$$

The apparent power at presence high harmonics define through effective values of non-harmonic voltage and current

$$S = UI = \sqrt{U^2 + \sum_{k=1}^{\infty} U_k^2}\sqrt{I_0^2 + \sum_{k=1}^{\infty} I_k^2} = \sqrt{\left(U_0^2 + \sum_{k=1}^{\infty} U_k^2\right)\left(I_0^2 + \sum_{k=1}^{\infty} I_k^2\right)}$$

This power turns out to be more, than definite through values of active and reactive powers component

$$S > \sqrt{\left(U_0^2 + \sum_{k=1}^{\infty} U_k^2\right)\left(I_0^2 + \sum_{k=1}^{\infty} I_k^2\right)} = \sqrt{P^2 + Q^2} .$$

This equality is fulfiled only in circuits in which the form non-sinusoidal currents and voltages completely identical.

The value

$$T = \sqrt{S^2 - P^2 - Q^2}$$

is called as the distortion power. Quotient T/S characterizes the distinguishing degree in the forms of curves current and voltage.

2.2. Study guides as to the calculation of three-phase non-harmonic circuits

1. If in three-phase circuit acts symmetrical three-phase non-sinusoidal supply power, then in this case EMF have similar form and shift in time on the one third period of repetition

$$e_A = \sum_{k=1}^{\infty} E_{km} \sin k\omega t;\ e_B = \sum_{k=1}^{\infty} E_{km} \sin k(\omega t - 2\pi/3);\ e_C = \sum_{k=1}^{\infty} E_{km} \sin k(\omega t + 2\pi/3).$$

The harmonic components with numbers *κ=1,4,7,10* and so on will form symmetrical systems of positive sequence.

The harmonic components with numbers *κ=2,5,8,11* and so on will form symmetrical systems of negative sequence.

The harmonic components with numbers *κ=3,6,9,12* and so on (this гармоники aliquot three) will form symmetrical systems of zero sequence.

Peculiarity aliquot three harmonic components is that they coincide as to phase. This circumstance brings to the peculiarities of the three-fase circuits work in presence high harmonic components, that there is necessary to take into account in calculation.

2. At the connection of the three-phase power supply clamps by triangle in the regime of open circuit in windings flow current, conditional third and aliquot three harmonic components. The effective value of such current is determined as $I = \sqrt{\sum_{k=3,6,9,...}^{\infty} I_k^2}$.

3. At the connection of the clamps of three-phase power supply by open triangle that in the point of triangle discontinuity acts voltage

$u = \sum_{k=3,6,9,...}^{\infty} 3E_{km} \sin k(\omega t + \psi_k)$. The effective value of such voltage is determined as

$$U = 3\sqrt{\sum_{k=3,6,9,...}^{\infty} U_k^2}\ .$$

4. In linear voltage irrespective of the clamps connection scheme of power supply are absent harmonic components aliquot three.

At connection by star:

- phase voltage effective value $U_P = \sqrt{\sum_{k=1,3,5,7,9,...}^{\infty} U_k^2}$.
- line voltage effective value $U_L = \sqrt{3}\sqrt{\sum_{k=1,5,7,...}^{\infty} U_k^2}$.
- the quotient of linear volage to phase one

$$\frac{U_L}{U_P} = \sqrt{3}\,\frac{\sqrt{\sum\limits_{k=1,5,7,...}^{\infty} U_k^2}}{\sqrt{\sum\limits_{k=1,3,5,7,9,...}^{\infty} U_k^2}} < \sqrt{3}\,.$$

At connection by triangle voltage component, due to harmonic components aliquot three, neither will appear between the phase clamps, because will be recompensed the voltage drop on inner phase impedances. If the current in the triangle side comprises harmonic components, aliquot three $I_P = \sqrt{\sum\limits_{k=1,3,5,6,7,9,...}^{\infty} I_k^2}$, that into exterior (linear) wire the current will not comprise harmonic components, aliquot three $I_L = \sqrt{3}\sqrt{\sum\limits_{k=1,5,7,...}^{\infty} I_k^2}$.

5. At the connection of power supply and symmetrical charge by star and absence neutral wire the linear current will not comprise harmonic components, aliquot three, i.e. $I_L = \sqrt{\sum\limits_{k=1,5,7,...}^{\infty} I_k^2}$.

Between the null points of power supply and load acts the voltage $U_{Nn} = \sqrt{\sum\limits_{k=3,6,9,...}^{\infty} U_k^2}$.

2.2 The circuit parameters calculation of single-phase non-harmonic at resistance-capacitance in scheme

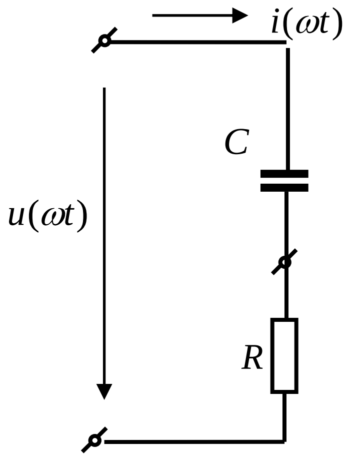


Fig. 2.1

Task.

To electric circuit, scheme which is presented on Fig. 2.1, is applied non-sinusoidal periodically voltage

$u(\omega t) = 68{,}8\sin\omega t + 7{,}64\sin 3\omega t + 2{,}75\sin(5\omega t - 180^0)$,V.

Scheme parameters $R = 5$ ohm, $x_C = \frac{1}{\omega C} = 15$ ohm.

Determine instantaneous current value in circuit, effectiv value of one and the circuit's power factor.

Task solving.

We determine of impedance for each of harmonic components:

-for the first harmonic component $\underline{Z}_1 = R - jx_C = 5 - j15 = 15{,}81e^{-j71^0}\,\Omega$;

- for the third harmonic component $\underline{Z}_3 = R - jx_C = 5 - j5 = 7{,}07e^{-j45^0}\,\Omega$;

- for the fifth harmonic component $\underline{Z}_5 = R - jx_C = 5 - j3 = 5{,}83e^{-j31^0}\,\Omega.$

We reckon amplitude of current harmonic components phasor values:

- the first current harmonic component $\underline{I}_{1m} = \dfrac{68{,}8}{15{,}81e^{-j71^0}} = 4{,}35e^{j71^0}\,A;$

- the third current harmonic component $\underline{I}_{3m} = \dfrac{7{,}64}{7{,}07e^{-j45^0}} = 1{,}08e^{j45^0}\,A;$

- the fifth current harmonic component $\underline{I}_{5m} = \dfrac{2{,}75e^{-j180^0}}{5{,}83e^{-j31^0}} = 0{,}471e^{-j149^0}\,A..$

Instantaneous current value:

$$i(\omega t) = 4{,}35\sin(\omega t + 71^0) + 1{,}08\sin(3\omega t + 45^0) + 0{,}471\sin(5\omega t - 149^0),\,A$$

The circuit active power

$$P = U_0 I_0 + \sum_{k=1}^{\infty} U_k I_k \cos\varphi_k = \frac{68{,}8 \cdot 4{,}35}{2}\cos 71^0 +$$

$$+ \frac{7{,}64 \cdot 1{,}08}{2}\cos 45^0 + \frac{2{,}75 \cdot 0{,}471}{2}\cos 31^0 = 52{,}19Wt.$$

The circuit reactive power

$$Q = \sum_{k=1}^{\infty} U_k I_k \sin\varphi_k = \frac{68{,}8 \cdot 4{,}35}{2}\sin 71^0 +$$

$$+ \frac{7{,}64 \cdot 1{,}08}{2}\sin 45^0 + \frac{2{,}75 \cdot 0{,}471}{2}\sin 31^0 = 144{,}73\,\text{var}.$$

Effective values of non-sinusoidal voltage and current

$$U = \sqrt{U_0^2 + \sum_{k=1}^{\infty} U_k^2} = \sqrt{\frac{68{,}8^2}{2} + \frac{7{,}64^2}{2} + \frac{2{,}75^2}{2}} = 49V;$$

$$I = \sqrt{I_0^2 + \sum_{k=1}^{\infty} I_k^2} = \frac{1}{\sqrt{2}}\sqrt{4{,}35^2 + 1{,}08^2 + 0{,}471^2} = 3{,}18A.$$

The circuit apparent power

$$S = UI = 49 \cdot 3{,}18 = 155{,}82VA.$$

The distortion power

$$T = \sqrt{S^2 - P^2 - Q^2} = \sqrt{155{,}82^2 - 52{,}19^2 - 144{,}73^2} = 24{,}68\,\text{var}.$$

The circuit's power factor

$$\chi = \frac{P}{S} = \frac{52{,}19}{155{,}82} = 0{,}33.$$

2.3. The single-phase non-harmonic circuit parameters calculation at series connection resistance-inductance-capacitance in scheme

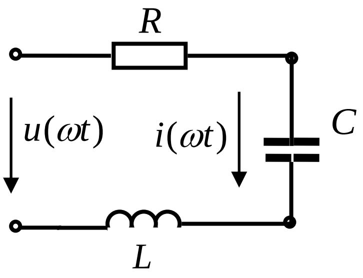


Fig.2.2

Task.

Determine current in series circuit, Fig.2.2, which has parameters $R=10$ ohm, $L=0{,}05$ H $C=22{,}5\cdot10^{-6}$ F. The which applied to the scheme clamps is non-sinusoidally

$u(\omega t)=180\sin\omega t+60\sin3\omega t+40\sin(5\omega t+0{,}1\cdot\pi),V$.

Angular frequency of main harmonic $\omega=314 rad/s.$

Task solving.

We determine the circuit impedance and the phase drift corners for every harmonic component:

- the first harmonic component

$$Z_1=\sqrt{R^2+(\omega L-1/\omega C)^2}=\sqrt{10^2+(314\cdot0{,}05-1/314\cdot22{,}5\cdot10^{-6})^2}=126\text{Îì};$$

$$tg\varphi_1=\frac{\omega L-1/\omega C}{R}=\frac{314\cdot0{,}05-1/314\cdot22{,}5\cdot10^{-6}}{10}=-12{,}53;\ \varphi_1=-85{,}3^0.$$

- the third harmonic component

$$Z_3=\sqrt{R^2+(3\omega L-1/3\omega C)^2}=\sqrt{10^2+(3\cdot314\cdot0{,}05-1/3\cdot314\cdot22{,}5\cdot10^{-6})^2}=10\text{Îì};$$

$$tg\varphi_3=\frac{3\omega L-1/3\omega C}{R}=\frac{3\cdot314\cdot0{,}05-1/3\cdot314\cdot22{,}5\cdot10^{-6}}{10}=0;\ \varphi_3=0^0.$$

- the fifth harmonic component

$$Z_5=\sqrt{R^2+(5\omega L-1/5\omega C)}=\sqrt{10^2+(5\cdot314\cdot0{,}05-1/5\cdot314\cdot22{,}5\cdot10^{-6})}=51{,}2\text{Îì};$$

$$tg\varphi_5=\frac{5\omega L-1/5\omega C}{R}=\frac{5\cdot314\cdot0{,}05-1/5\cdot314\cdot22{,}5\cdot10^{-6}}{10}=5{,}02;\ \varphi_5=78{,}7^0.$$

We reckon amplitudes of every current harmonic components

$$I_{1m}=U_{1m}/Z_1=130/126=1{,}43A;\ I_{3m}=U_{3m}/Z_3=60/10=6A;$$

$$I_{5m}=U_{5m}/Z_5=40/51{,}2=0{,}78A.$$

The effective value current in circuit

$$I=\sqrt{\frac{I_{1m}^2}{2}+\frac{I_{3m}^2}{2}+\frac{I_{5m}^2}{2}}=\frac{1}{\sqrt{2}}\sqrt{1{,}43^2+6^2+0{,}78^2}=6{,}2A.$$

Instantaneous current value:

$$i(\omega t)=i_1(\omega t)+i_3(3\omega t)+i_5(5\omega t)=1{,}43\sin(\omega t-85{,}3^0)+$$

$$+16\sin(3\omega t)+0{,}78\sin(5\omega t+78{,}7^0),A.$$

There is necessary to notice that for third harmonic component is seen the voltage resonance. The impedance for third harmonic component is equal to ohmic resistance of circuit. Third harmonic component has essential specific weight in current curve, than in voltage curve. The correlation of amplitudes third to the first harmonic components

$$\frac{U_{3m}}{U_{1m}}=\frac{60}{180}=0{,}33;\ \frac{I_{3m}}{I_{1m}}=\frac{6}{1{,}43}=4{,}2.$$

2.4. The single-phase non-harmonic circuit parameters calculation at mixed connection of resistive-inductive-capacitive resistance in scheme

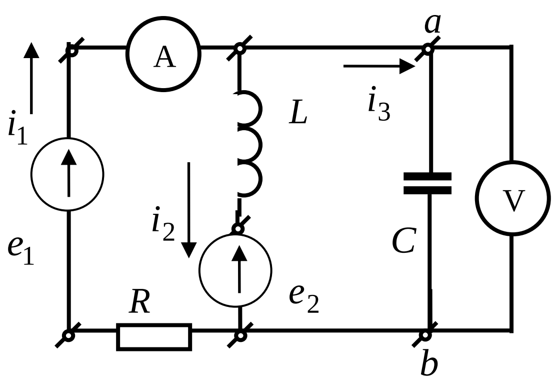


Fig. 2.3

Task.
Given the scheme parameters on main harmonic component (Fig. 2.3) $R=4$ ohm, $x_L=\omega L=3$ ohm, $x_C=\frac{1}{\omega C}=12$ ohm.
Into circuit included two sources of energy supplies: non-sinusoidal one

$$e(\omega t)=4+34\sin\omega t+12\sin(2\omega t+\frac{\pi}{9}),V$$

and direction one $e_2(\omega t)=12V$.
Determine instantaneous current values in circuit branches and reading of an electromagnetic system devices.

Task solving.
The calculation we perform by the superposition method from action every EMF harmonic component with symbolic method using.
For the EMF direct components current value

$$I_{10}=I_{20}=\frac{e_{10}-e_{20}}{R}=\frac{4-12}{4}=-12;I_{30}=0.$$

We shall determine branches current from action the first EMF harmonic component. The circuit input impedance for the first harmonic component

$$\underline{Z}_1=R+\frac{jx_L\cdot(-jx_C)}{jx_L-jx_C}=4+\frac{j3\cdot(-j12)}{j3-j12}=4-j4=5{,}661e^{j45^0}\text{Îì};$$

The phaser current amplitude value in circuit unbranched part

$$\underline{I}_{m11}=\frac{\underline{E}_{m1}}{\underline{Z}_1}=\frac{34}{5{,}66e^{j45^0}}=6e^{-j45^0}A;$$

The instantaneous current value in circuit unbranched part

$i_{11}(\omega t) = 6\sin(\omega t - 45^0), A.$

The current of second branch

$$\underline{I}_{m21} = \underline{I}_{m11} \frac{-j\dfrac{1}{\omega C}}{j\omega L - j\dfrac{1}{\omega C}} = 6e^{-j45^0} \frac{-j12}{-j9} = 8e^{-j45^0} A;$$

$i_{21}(\omega t) = 8\sin(\omega t - 45^0), A.$

The current in third branch

$$\underline{I}_{m31} = \underline{I}_{m11} \frac{j\omega L}{j\omega L - j\dfrac{1}{\omega C}} = 6e^{-j45^0} \frac{-j3}{-j9} = -2e^{-j45^0} A;$$

$i_{31}(\omega t) = -2\sin(\omega t - 45^0), A.$

For the second harmonic component circuit impedance and current

$$\underline{Z}_2 = R + \frac{2jx_L \cdot \left(\dfrac{-jx_C}{2}\right)}{2jx_L - \dfrac{jx_C}{2}} = 4 + \frac{2j3 \cdot \left(-\dfrac{j12}{2}\right)}{2j3 - j\dfrac{12}{2}} = 4 + \infty = \infty, \Omega;$$

$$\underline{I}_{m21} = \frac{\underline{E}_{m2}}{\underline{Z}_2} = \frac{12}{\infty} = 0A;$$

The current in circuit unbranched part $i_{12}(2\omega t) = 0, A.$

As to second harmonic component in circuit is seen current resonance.

Current in branches from second harmonic component

$$\underline{I}_{m22} = \frac{\underline{E}_{m2}}{\underline{Z}_{22}} = \frac{12e^{j20^0}}{2j3} = 2e^{-j70^0} A;$$

$i_{22}(2\omega t) = 2\sin(2\omega t - 70^0), A.$

$$\underline{I}_{m32} = \frac{\underline{E}_{m2}}{\underline{Z}_{32}} = \frac{12e^{j20^0}}{-j\dfrac{12}{2}} = 2e^{j110^0} A;$$

$i_{32}(2\omega t) = 2\sin(2\omega t + 110^0), A.$

Note: the first index shows the number of district, and second shows the harmonic component number.

We find the electromagnetic system's gage readings (non-sinusoidal currents and voltages effective values). The electromagnetic system ammeter indications

$$I = \sqrt{I_0^2 + \frac{I_{m11}^2 + I_{m12}^2}{2}} = \sqrt{2^2 + \frac{6^2 + 0^2}{2}} = 4{,}7A.$$

The electromagnetic system voltmeter indications

$$U_{ab} = \sqrt{U_{ab0}^2 + U_{ab1}^2 + U_{ab2}^2} = \sqrt{12^2 + 17^2 + 8{,}51^2} = 22{,}5;$$

$$U_{ab0} = e_2 = 12V;$$

$$U_{ab1}=I_{11}Z_{ab1}=\frac{6}{\sqrt{2}}4=17V;\; U_{ab2}=I_{12}Z_{ab2}=\frac{E_{m2}}{\sqrt{2}}=\frac{12}{\sqrt{2}}=8{,}51V.$$

The instantaneous current value in the circuit branches

Current in circuit unbranched part

$$i_1(\omega t)=i_{10}+i_{11}+i_{12}=-2+6\sin(\omega t-45^0)+0, A.$$

Current in second branch

$$i_2(\omega t)=i_{20}+i_{21}+i_{22}=-2+8\sin(\omega t-45^0)+2\sin(2\omega t-70^0), A.$$

Third branch current

$$i_3(\omega t)=i_{30}+i_{31}+i_{32}=-2-2\sin(\omega t-45^0)+2\sin(2\omega t+110^0), A.$$

2.5. The three-phase non-harmonic circuit parameters calculation at the load gather in symmetrical delta-connection

Task.

In three-phase circuit there is non-harmonic EMFs symmetrical system (Fig.2.4). The EMF in B phase of three-phase symmetrical non-harmonic source is known

$$e_{BN}(\omega t)=60\sqrt{2}\sin(\omega t+\frac{\pi}{6})+45\sqrt{2}\sin(3\omega t)+30\sqrt{2}\sin(5\omega t-30^0), V.$$

Impedances in the load phases on frequency of the first harmonic component is known $Z_{ab1}=Z_{bc1}=Z_{ca1}=15, Ohm$.

Determine the electromagnetic system's gage readings (non-sinusoidal currents and voltages effective values).

Task solving.

The voltmeter indicate linear voltage effective value from the first and fifth harmonic components, because third harmonic components will form voltage zero sequence in three-phase source

$$\underline{U}_{AB3}=\underline{U}_{A3}-\underline{U}_{B3}=0:$$

$$U_L=\sqrt{3}\sqrt{U_{P1}^2-U_{P2}^2}=\sqrt{3}\sqrt{60^2-30^2}=116V.$$

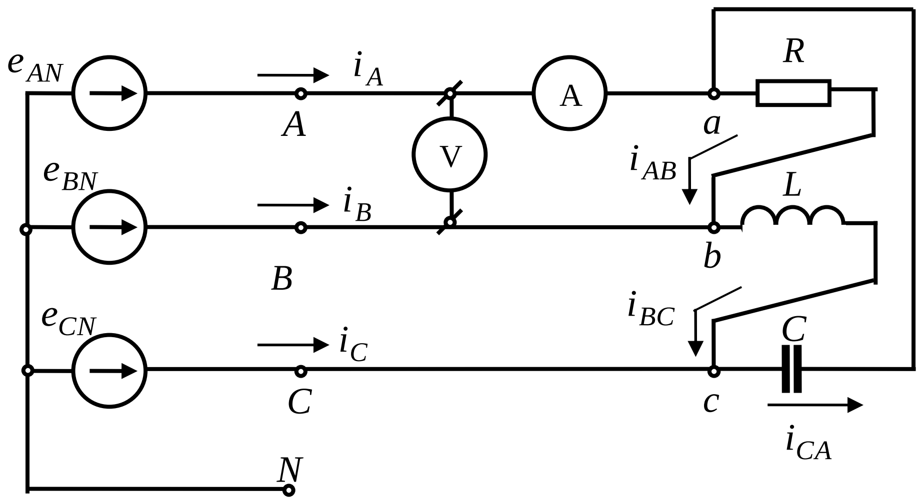


Fig. 2.4

The ammeter indications

$I=\sqrt{I_{A1}^2+I_{A5}^2};I_{A1}^2=0,$

where $\underline{I}_{A1}=\underline{I}_{AB1}-\underline{I}_{CA1}$; $\underline{I}_{A5}=\underline{I}_{AB5}-\underline{I}_{CA5}$; $\underline{I}_{AB1}=\dfrac{\underline{U}_{AB1}}{\underline{Z}_{AB1}}$; $\underline{I}_{CA1}=\dfrac{\underline{U}_{CA1}}{\underline{Z}_{CA1}}$;

$\underline{I}_{AB5}=\dfrac{\underline{U}_{AB5}}{\underline{Z}_{AB5}}$; $\underline{I}_{CA5}=\dfrac{\underline{U}_{CA5}}{\underline{Z}_{CA5}}$.

First of all we shall determine the phasor linear voltages for the first harmonic component in given phase EMF

$e_{BN1}=u_{BN1}=60\sqrt{2}\sin(\omega t+\dfrac{\pi}{6}),V;\ \underline{U}_{BN1}=60e^{j30^0};$

$\underline{U}_{AN1}=\underline{U}_{BN1}e^{j120^0}=60e^{j30^0}e^{j120^0}=60e^{j150^0},V;$

$\underline{U}_{CN1}=\underline{U}_{AN1}e^{-j120^0}=60e^{j30^0}e^{-j120^0}=60e^{-j90^0},V;$

$\underline{U}_{AB1}=\underline{U}_{AN1}-\underline{U}_{BN1}=\sqrt{3}\underline{U}_{AN1}e^{j30^0}=\sqrt{3}\cdot 60e^{j150^0}e^{j30^0}=$

$=\sqrt{3}\cdot 60e^{j180^0}=-\sqrt{3}\cdot 60,V;$

$\underline{U}_{CA1}=\underline{U}_{CN1}-\underline{U}_{AN1}=\sqrt{3}\underline{U}_{CN1}e^{j30^0}=\sqrt{3}\cdot 60e^{-j90^0}e^{j30^0}=$

$=\sqrt{3}\cdot 60e^{-j60^0},V.$

We calculate linear voltage value for fifth harmonic component:

$e_{BN5}=u_{BN5}=30\sqrt{2}\sin(5\omega t-30^0),V;\ \underline{U}_{BN5}=30e^{-j30^0};$

$\underline{U}_{AN5}=\underline{U}_{BN5}e^{j120^0}=30e^{-j30^0}e^{-j120^0}=60e^{-j150^0},V;$

$\underline{U}_{CN5}=\underline{U}_{BN5}e^{j120^0}=30e^{-j30^0}e^{j120^0}=30e^{j90^0},V;$

$\underline{U}_{AB5}=\underline{U}_{AN5}-\underline{U}_{BN5}=\sqrt{3}\underline{U}_{AN5}e^{-j30^0}=\sqrt{3}\cdot 30e^{-j150^0}e^{j30^0}=$

$=\sqrt{3}\cdot 30e^{-j180^0}=-\sqrt{3}\cdot 30,V;$

$$\underline{U}_{CA5} = \underline{U}_{CN1} - \underline{U}_{AN1} = \sqrt{3}\underline{U}_{CN5}e^{-j30^0} = \sqrt{3}\cdot 30e^{j90^0}e^{-j30^0} =$$
$$= \sqrt{3}\cdot 30e^{j60^0}, V.$$

The phase currents of first harmonic component:

$$\underline{I}_{AB1} = \frac{\underline{U}_{AB1}}{\underline{Z}_{AB1}} = \frac{-\sqrt{3}\cdot 60}{15} = -4\sqrt{3} = 4\sqrt{3}e^{j180^0}, A;$$

$$\underline{I}_{CA1} = \frac{\underline{U}_{CA1}}{\underline{Z}_{CA1}} = \frac{-\sqrt{3}\cdot 60e^{j60^0}}{-j15} = 4\sqrt{3}e^{j30^0}, A.$$

The line currents of first harmonic component:

$$\underline{I}_{A1} = \underline{I}_{AB1} - \underline{I}_{CA1} = -4\sqrt{3} - 4\sqrt{3}e^{j30^0} = -8{,}92 - j3{,}46 = 9{,}56e^{-j158^0}, A.$$

The phase currents of fifth harmonic component:

$$\underline{I}_{AB5} = \frac{\underline{U}_{AB5}}{\underline{Z}_{AB5}} = \frac{-\sqrt{3}\cdot 30}{15} = -2\sqrt{3} = 2\sqrt{3}e^{j180^0}, A;$$

$$\underline{I}_{CA5} = \frac{\underline{U}_{CA5}}{\underline{Z}_{CA5}} = \frac{-\sqrt{3}\cdot 60e^{j60^0}}{-j\frac{15}{5}} = 10\sqrt{3}e^{j150^0} = -15 + j8{,}65, A.$$

$$\underline{I}_{A5} = \underline{I}_{AB5} - \underline{I}_{CA5} = -2\sqrt{3} - 10\sqrt{3}e^{j150^0} = 11{,}54 - j8{,}65 = 14{,}42e^{-j36{,}8^0}, A.$$

The electromagnetic system ammeter indication

$$I = \sqrt{I_{A1}^2 + I_{A5}^2} = \sqrt{9{,}57^2 + 14{,}42^2} = 17{,}3, A.$$

2.6. The three-phase non-harmonic circuit parameters calculation at the load gather in symmetrical Y-connection with the neutral wire

Task.

In three-phase circuit there is non-harmonic EMFs symmetrical system (Fig.2.5). The phase B EMF of three-phase symmetrical non-harmonic source is known

$$e_{BN}(\omega t) = 100\sqrt{2}\sin(\omega t + 30^0) + 60\sqrt{2}\sin(3\omega t - 60^0) + 30\sqrt{2}\sin(5\omega t - 100^0), V.$$

Impedances in the load phases on frequency of the first harmonic component is known $Z_{ab1} = Z_{bc1} = Z_{ca1} = 15, Ohm$.

Write down the instantaneous value of linear voltage u_{BC} and current in neutral wire i_{Nn}. Determine indications of the electrodynamic system devices.

Task solving.

The equation of linear voltage u_{BC1} for the first harmonic component we find through difference fitting phase voltages

$u_{B1} = 100\sqrt{2}\sin(\omega t + 30^0)$; $\underline{U}_{B1} = 100e^{j30^0}$;

$u_{C1} = 100\sqrt{2}\sin(\omega t - 90^0)$; $\underline{U}_{C1} = 100e^{-j90^0}$.

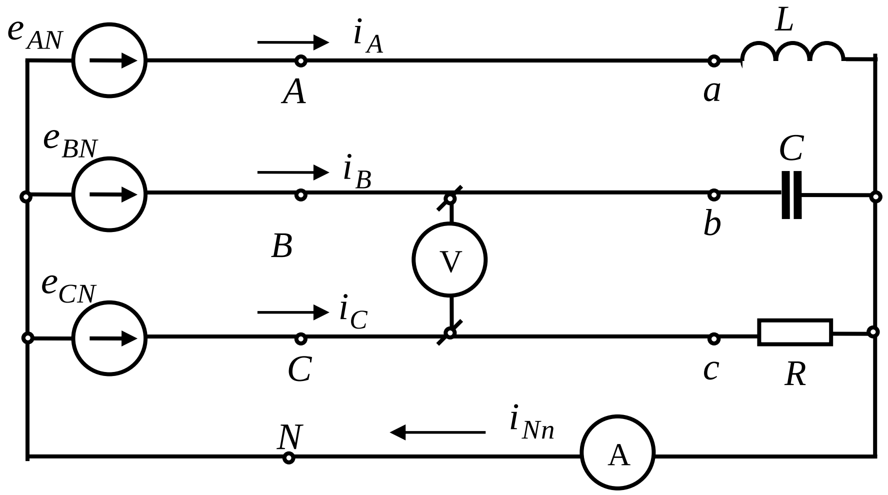


Fig. 2.5

The line voltage

$\underline{U}_{BC1} = \underline{U}_{B1} - \underline{U}_{C1} = 100(e^{j30^0} - e^{-j90^0}) = 173e^{j60^0}, V;$

The instantaneous value

$u_{BC1} = \sqrt{2} \cdot 173 \cdot \sin(\omega t + 60^0), V.$

For the third harmonic component because of zero sequence we have

$u_{BC3} = u_{B3} - u_{B3} = 0.$

The equation of linear voltage u_{BC5} for the fifth harmonic component we find through difference fitting phase voltages

$u_{B5} = 30\sqrt{2}\sin(5\omega t - 100^0);\ \underline{U}_{B5} = 30e^{-j100^0};$

$u_{C5} = 30\sqrt{2}\sin(5\omega t + 20^0);\ \underline{U}_{C5} = 100e^{j20^0};$

The line voltage

$\underline{U}_{BC5} = \underline{U}_{B5} - \underline{U}_{C5} = \sqrt{3} \cdot 30 \cdot e^{-j100^0} e^{-j30^0} = \sqrt{3} \cdot 30 \cdot e^{-j130^0}, V;$

The instantaneous value

$u_{BC5} = \sqrt{2} \cdot \sqrt{3} \cdot 30 \cdot \sin(5\omega t - 130^0), V.$

The instantaneous value of line voltage contain the first and fifth harmonic components

$u_{BC} = u_{BC1} + u_{BC5} = \sqrt{2} \cdot 173 \cdot \sin(\omega t + 60^0) + \sqrt{2} \cdot \sqrt{3} \cdot 30 \cdot \sin(5\omega t - 130^0), V.$

The electrodynamic system voltmeter indication

$U_{BC} = \sqrt{U_{BC1}^2 + U_{BC5}^2} = \sqrt{173^2 + (\sqrt{3} \cdot 30)^2} = 180, V.$

We shall find current equation in neutral wire.

Phase voltage phasor value for the first harmonic component

$\underline{U}_{A1} = \underline{U}_{B1} e^{j120^0} = 100e^{j30^0} e^{j120^0} = 100e^{j150^0};\ \underline{U}_{B1} = 100e^{j30^0};$

$\underline{U}_{C1} = \underline{U}_{B1} e^{-j120^0} = 100e^{j30^0} e^{-j120^0} = 100e^{-j90^0};$

Currents phasor value in phases and neutral wire for the first harmonic component

$$\underline{I}_{A1}=\frac{\underline{U}_{A1}}{\underline{Z}_{A1}}=\frac{100e^{j150^0}}{j15}=6{,}66e^{j60^0},A;\ \underline{I}_{B1}=\frac{\underline{U}_{B1}}{\underline{Z}_{B1}}=\frac{100e^{j30^0}}{j15}=6{,}66e^{-j60^0},A;$$

$$\underline{I}_{C1}=\frac{\underline{U}_{C1}}{\underline{Z}_{C1}}=\frac{100e^{-j0^0}}{15}=6{,}66e^{-j90^0},A.$$

$$\underline{I}_{Nn1}=\underline{I}_{A1}+\underline{I}_{B1}+\underline{I}_{C1}=6{,}66e^{j60^0}+6{,}66e^{-j60^0}+6{,}66e^{-j90^0}=13{,}33e^{-j30^0},A.$$

Currents phasor value in phases and neutral wire for the third harmonic component

$$\underline{U}_{A3}=\underline{U}_{B3}=\underline{U}_{C3}=60e^{-j60^0},V.$$

$$\underline{I}_{A3}=\frac{\underline{U}_{A3}}{\underline{Z}_{A3}}=\frac{60e^{-j60^0}}{3j15}=1{,}33e^{-j150^0},A;\ \underline{I}_{B3}=\frac{\underline{U}_{B3}}{\underline{Z}_{B3}}=\frac{60e^{-j60^0}}{j15/3}=12e^{j30^0},A;$$

$$\underline{I}_{C3}=\frac{\underline{U}_{C3}}{\underline{Z}_{C3}}=\frac{60e^{-j60^0}}{15}=4e^{-j60^0},A.$$

$$\underline{I}_{Nn3}=\underline{I}_{A3}+\underline{I}_{B3}+\underline{I}_{C3}=1{,}33e^{-j150^0}+12e^{j30^0}+4e^{-j60^0}=11{,}38e^{j9{,}5^0},A.$$

Currents phasor value in phases and neutral wire for the fifth harmonic component

$$\underline{U}_{A5}=\underline{U}_{B5}e^{-j120^0}=30e^{-j100^0}e^{-j120^0}=30e^{-j220^0},V;\ \underline{U}_{B5}=30e^{-j100^0},V;$$

$$\underline{U}_{C5}=30e^{j20^0},V.$$

$$\underline{I}_{A5}=\frac{\underline{U}_{A5}}{\underline{Z}_{A5}}=\frac{30e^{-j220^0}}{5j15}=0{,}4e^{-j310^0},A;\ \underline{I}_{B5}=\frac{\underline{U}_{B5}}{\underline{Z}_{B5}}=\frac{30e^{-j100^0}}{j15/5}=10e^{-j10^0},A;$$

$$\underline{I}_{C5}=\frac{\underline{U}_{C5}}{\underline{Z}_{C5}}=\frac{30e^{-j20^0}}{15}=2e^{j20^0},A.$$

$$\underline{I}_{Nn5}=\underline{I}_{A5}+\underline{I}_{B5}+\underline{I}_{C5}=0{,}4e^{-j310^0}+10e^{-j10^0}+2e^{j20^0}=12e^{-j3{,}4^0},A..$$

The instantaneous value of current in neutral wire i_{Nn}

$$i_{Nn}=i_{Nn1}+i_{Nn3}+i_{Nn5}+\underline{I}_{B5}+\underline{I}_{C5}=\sqrt{2}\cdot 13{,}33\sin(\omega t-30^0)+$$
$$+\sqrt{2}\cdot 11{,}38\sin(3\omega t+9{,}5^0)+\sqrt{2}\cdot 12\sin(5\omega t-3{,}4^0),A.$$

The indication of the electrodynamic system ammeter

$$I_{Nn}=\sqrt{I_{Nn1}^2+I_{Nn3}^2+I_{Nn5}^2}=\sqrt{13{,}33^2+11{,}38^2+12^2}=21{,}24,A.$$

2.7. The three-phase non-harmonic circuit parameters calculation at the load gather in symmetrical delta-connection and when there are impedances in lines

Task. In three-phase circuit there is non-harmonic EMFs symmetrical system (Fig.2.5). The phase B EMF of three-phase symmetrical non-harmonic source is known

In three-phase electric circuit acts symmetrical non-nsinusoidal system EMF: $e_{AB}(\omega t) = 140\cos(\omega t) + 60\sin(5\omega t), V$, where $\omega = 2\pi / T$ at *T=0,015 s*. The scheme parameters $R = 24$ ohm, $L = 7{,}5$ mH, $C = 37{,}5\,\mu F$. The scheme is presented on Fig. 2.6.
Determine the electromagnetic systems indications of ammeter and voltmeter and the active and apparent powers of three-phase system.

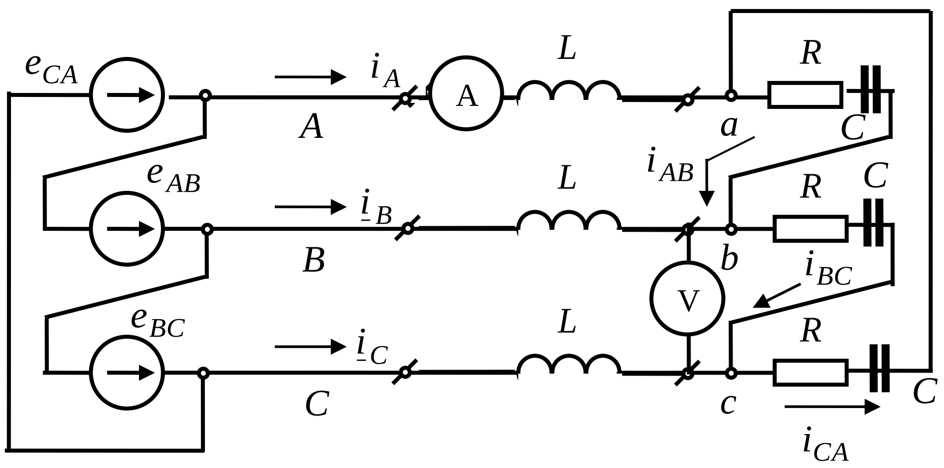


Fig. 2.6

Task solving.
For accounting the line inductive resistance we shall transform connections by the triangle of EMF power supply and load into equivalent stars, Fig.2.7. Find the phase voltage in the phases power supply on equivalent star
$u_{AB}(\omega t) = -140\cos(\omega t) - 60\sin(5\omega t), V,$
where $\omega = 2\pi / T = 2\pi / 0{,}015 = 418{,}6 rad / s$;

$$u_A(\omega t) = -\frac{140}{\sqrt{3}}\sin(\omega t - 120^0) + \frac{60}{\sqrt{3}}\sin(5\omega t 30^0) =$$

$$= -80{,}9\sin(\omega t - 120^0) + 34{,}7\sin(5\omega t + 30^0), V.$$

At the defining of phase voltage $u_A(\omega t)$ according to known line voltages, is taken into account, that they differ as to modulus in $\sqrt{3}$ time. Between phase and line voltages there is the shift phase 30^0 and for line voltage fifth harmonic component is seen the phase interlacing negative sequences. That is why the fifth harmonic component phase voltage lead line one on 30^0.

$$u_B(\omega t) = -80{,}9\sin(\omega t - 240^0) + 34{,}7\sin(5\omega t + 150^0) =$$

$$= 80{,}9\sin(\omega t - 60^0) + 34{,}7\sin(5\omega t + 150^0), V;$$

$$u_C(\omega t) = -80{,}9\sin(\omega t^0) + 34{,}7\sin(5\omega t - 90^0) =$$

$$= 80{,}9\sin(\omega t - 180^0) + 34{,}7\sin(5\omega t - 90^0), V.$$

The every line impedance:
- for the first harmonic component $\underline{Z}_{l1} = j\omega L = j418{,}6 \cdot 7{,}5 \cdot 10^{-3} = j3{,}14$ ohm;

- for the fifth harmonic component $\underline{Z}_{l5} = j5 \cdot 418{,}6 \cdot 7{,}5 \cdot 10^{-3} = j15{,}7$ ohm.

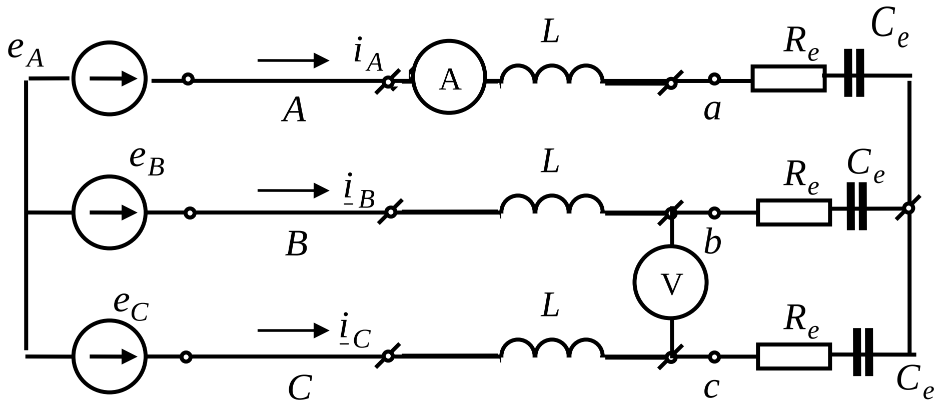


Fig. 2.7

Phase load impedance

$$\underline{Z}_A = R - jx_c; x_{C1} = \frac{1}{\omega_1 C} = \frac{1}{418{,}6 \cdot 37{,}5 \cdot 10^{-6}} = 63{,}7, \Omega;$$

$$x_{C5} = \frac{x_{C1}}{5} = \frac{63{,}7}{5} = 12{,}74, \Omega;$$

$$\underline{Z}_{\Delta 1} = 24 - j63{,}7; \underline{Z}_{P\Delta 5} = 24 - j12{,}74, \Omega.$$

For the equivalent star scheme the phase load impedance:

$$\underline{Z}_{Y1} = \frac{\underline{Z}_{\Delta 1}}{3} = \frac{24 - j63{,}7}{3} = 8 - j21{,}23 = 22{,}68e^{-j69{,}3^0}, \Omega;$$

$$\underline{Z}_{Y5} = \frac{\underline{Z}_{\Delta 5}}{3} = \frac{24 - j12{,}74}{3} = 8 - j4{,}246 = 9{,}056e^{-j27{,}3^0}, \Omega.$$

For the equivalent star scheme the phase load impedance with a glance of line wire impedances:

$$\underline{Z}_1 = \underline{Z}_{l1} + \underline{Z}_{Y1} = j3{,}14 + 8 - j21{,}23 = 19{,}8e^{-j66{,}1^0}, \Omega;$$

$$\underline{Z}_5 = j15{,}7 + 8 - j4{,}246 = 13{,}96e^{j55^0}, \Omega.$$

The line current amplitudes:

- for the first harmonic component $I_{mA1} = I_{mB1} = I_{mC1} = \dfrac{U_{m1}}{Z_1} = \dfrac{80{,}9}{19{,}8} = 4{,}085, A;$

- for the fifth harmonic component $I_{mA5} = I_{mB5} = I_{mC5} = \dfrac{U_{m5}}{Z_5} = \dfrac{34{,}7}{13{,}96} = 2{,}485, A.$

The ammeter indication

$$I = \frac{1}{\sqrt{2}}\sqrt{I_{mA1}^2 + I_{mA5}^2} = \frac{1}{\sqrt{2}}\sqrt{4{,}083^2 + 2{,}485^2} = 3{,}37, A.$$

The phase voltage amplitude on loade:

$U_{mA1} = I_{mA1} Z_{Y1} = 4{,}085 \cdot 22{,}68 = 92{,}64, V;$

$U_{mA5} = I_{mA5} Z_{Y5} = 2{,}485 \cdot 9{,}056 = 55{,}9, V.$

Phase voltage effective value on load

$$U_{PY} = \frac{1}{\sqrt{2}} \sqrt{U_{mA1}^2 + U_{mA5}^2} = \sqrt{92{,}64^2 + 55{,}9^2} = 76{,}7, V.$$

The voltmeter indication

$U = \sqrt{3} \cdot U_{PY} = 1{,}73 \cdot 76{,}7 = 132{,}69, V.$

Three-phase circuit active and apparent powers

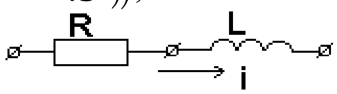
- active power $P = 3 \cdot I^2 \cdot R_{Y1} = 3 \cdot 3{,}4^2 \cdot 8 = 277{,}44 Wt.$
- apparent power $S = \sqrt{3} \cdot U_L \cdot I_L = \sqrt{3} \cdot 108 \cdot 3{,}4 = 635 VA,$

where non-sinusoidal voltage effective value

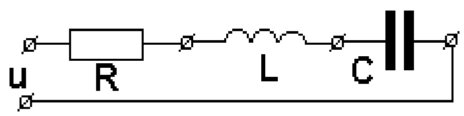
$$U_L = \sqrt{U^2{}_{L1} + U^2{}_{L5}} = \frac{1}{\sqrt{2}} \sqrt{140^2 + 60^2} = 108, V.$$

2.8. Questions for one's own checking as to the calculation methods of single-phase circuits within non-harmonic voltages and currents

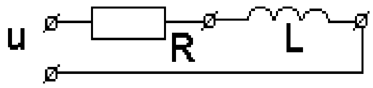
1. Find voltage U effective value if R=10 Ω, ωL=10 Ω, $i = \left(5 + 5\sqrt{2}\sin\omega t - 5\sqrt{2}\sin\left(2\omega t + 45^o\right)\right)$,A.



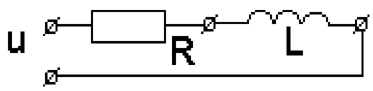
2. Find current I effective value, if $u = \left(100 \cdot \sqrt{2} \cdot \sin\omega t - 100\sqrt{2}\sin\left(3\omega t + 60^o\right)\right)$, B; $\omega L = 10\,\Omega$; $\dfrac{1}{\omega C}$ =30Ω.



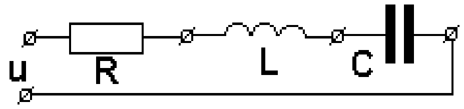
3. Find circuit power factor, if given R=4 Ω, $x_l = \omega \cdot L = 3$ Ω, and current is $i = \left(4 + 3\sqrt{2}\sin\omega \cdot t\right)$, A.



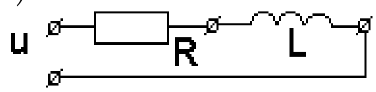
4. Find apparent power S of the circuit, if given: $i = (4 + 3\sqrt{2}\sin\omega t)$, A; $\underline{z} = (4 + j3)$, Ω.



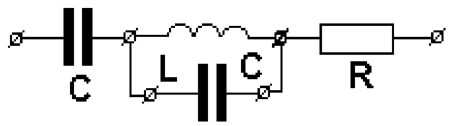
5. For the circuit applied voltage $u = \left(100 + 100\sqrt{2}\sin\left(100t + 45^o\right)\right)$ V. Find reactive power Q of the circuit, if given $R = \omega \cdot L = \dfrac{1}{\omega \cdot C} = 10$ Ω.



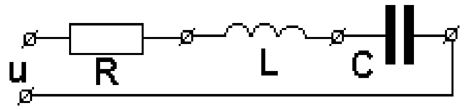
6. Determine distortion power T of passive one-port scheme, if given voltage $u = 100\sqrt{2}\sin\omega t$, V and current $i = \left(10 + 10\sqrt{2}\sin\left(\omega t + 60^o\right)\right)$A.
7. For the one-port scheme applied voltage $u = \left(100 + 141\sin\left(100t + 45^o\right)\right)$, V, under one flows current $i = 5\sin 100t$, A. Find apparent power S.
8. Determinate power factor of passive one-port scheme, if given $u = \left(120\sqrt{2}\sin\omega t + 50\sqrt{2}\sin\left(3\omega t + 45^o\right)\right)$, V, $i = 4\sqrt{2}\sin\omega t$, A.
9. Determinate active power P of circuit which contain series connected R, L elements if $i = \left(6 + 3\sqrt{2}\sin\omega t\right)$, A, R=4 Ω, ωL=3 Ω.



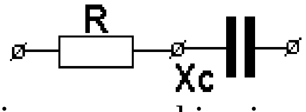
10. Find active power P which is consumed in circuit if $u = \left(100 + 100\sin\left(\omega t + 45^o\right)\right)$, V,
C=100 μF, R=10 Ω and circuit is work at current resonance (ω=314 rad/s)



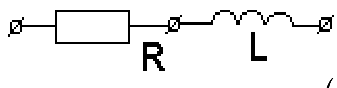
11. For the series connected R, L, C elements applied non-sinusoid voltage $u = 100 + \sqrt{2}\cdot 100\sin\left(100t + 45^o\right)$, V, $\omega L = \dfrac{1}{\omega C} = R$=100 Ω. Find active power P which is consumed in circuit.



12. Find active power P which is consumed in circuit if $u = \left(100\sqrt{2}\sin\omega t + 20\sqrt{2}\sin 3\omega t\right)$,V, R=10 Ω, $\dfrac{1}{\omega C}$=30 Ω.

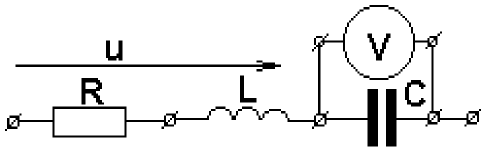


13. Find active power P which is consumed in circuit if $u = \left(100\sqrt{2}\sin\omega t + 40\sqrt{2}\sin 2\omega t\right)$,V, R=20 Ω, ω_L=10 Ω.

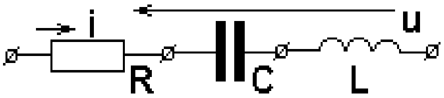


14. To the one-port scheme is applied voltage $u = \left(100 + 150\sin\left(100t + 45^o\right)\right)$,V, under one flows current i=5 A (instantaneous value). Find active power P which is consumed in the one-port scheme.

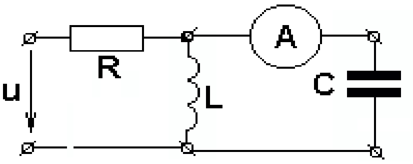
15. To the one-port scheme applied voltage $u=\left(100+141\sin\left(100t+45^{o}\right)\right)$, V, under one flows current $i=5\sin(100t)$, A. Find active power P this is consumed in the one-port scheme.
16. To the circuit applied non-sinusoidal voltage $u=100+5\sin\omega t$, V; $\frac{1}{\omega C}=\omega L=R$. Determine the reading of magnetoelectric system voltmeter.



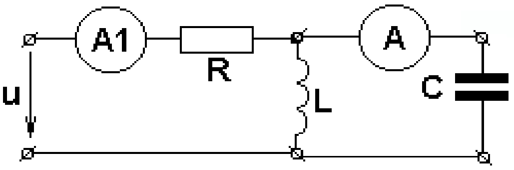
17. Current and voltage of one-port scheme are given $i=I_{m1}\sin\omega t$, A $u=U_o+U_{m1}\sin\left(\omega t+45^{o}\right)$, V. Define $x_L=\omega L$ if $R=1/(\omega C)=40\,\Omega$.



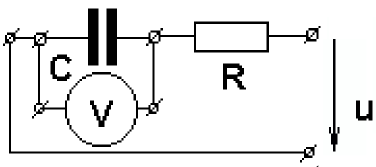
18. To the circuit apply non-sinusoidal voltage $u=\left(20+10\sqrt{2}\sin\omega t\right)$, B. Given R=10 Ω, $\omega L=\frac{1}{\omega C}$=10 Ω. Determine ammeter A readings of electromagnetic system.



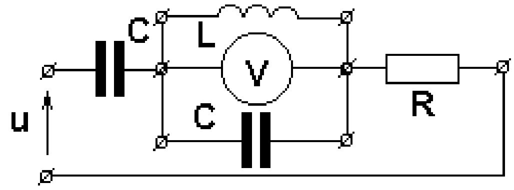
19. To the circuit applied non-sinusoidal voltage $u=\left(20+10\sqrt{2}\sin 100\omega t\right)$, V; R=10 Ω, $\omega L=\frac{1}{\omega C}$. Determine electromagnetic system ammeter A1 readings.



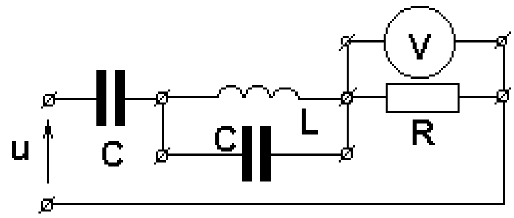
20. To the circuit applied non-sinusoidal voltage $u=\left(100+70{,}5\sin 100t\right)$,V, R=100 Ω, C=100 μF. Determine display of voltmeter V of electromagnetic system.



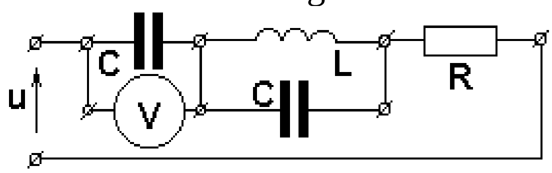
21. To circuit applied non-sinusoidal voltage $u=\left(100+141\sin 100\right)$, V. The circuit parameters given: C=100 μF, L=1 H, R=10 Ω. Determine electromagnetic system voltmeter V readings.



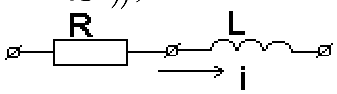
22. To circuit applied non-sinusoidal voltage $u = (100 + 150\sin\omega t)$, V. The circuit parameters are: ω=100 rad/s, C=100 μF, L=1 H, R=10 Ω. Determine electromagnetic system voltmeter V readings.



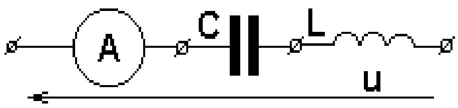
23.Given: $u = (100 + 150\sin 100t)$, V, C=100 μF, L=1 H, R=10 VΩ. Determine electromagnetic system voltmeter V readings.



24. Find voltage U effective value if R=20 Ω, ωL=5 Ω, $i = \left(5 + 5\sqrt{2}\sin\omega t - 5\sqrt{2}\sin\left(2\omega t + 45^o\right)\right)$,A.

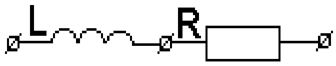


25. Determine electromagnetic system ammeter A readings, if $u = \left(100\sqrt{2}\sin\omega t - 100\sqrt{2}\sin\left(3\omega t + 60°\right)\right)$, V. ωL=10 ohm, $\dfrac{1}{\omega C}$=30 ohm.

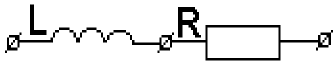


26. Find voltage that applied to circle, if there are given R=4 ohm, $x_L = \omega L$=3 ohm, and current $i = \left(4 + 3\sqrt{2}\sin\omega t\right)$, A.

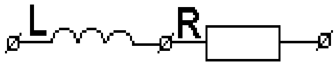
27. Determinate active power P of circuit which contain series connected R, L elements if R=4 ohm, $x_L = \omega L$=3 ohm, and current is $i = \left(4 + 3\sqrt{2}\sin\omega t\right)$, A.



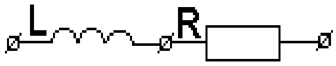
28. Determinate reactive power Q of circuit which contain series connected R, L elements if R=4 ohm, $x_L = \omega L$ =3 ohm, and current is $i = \left(4 + 3\sqrt{2}\sin \omega t\right)$, A.



29. Determinate apparent power S of circuit which contain series connected R, L elements if R=4 ohm, $x_L = \omega L$ =3 ohm, and current is $i = \left(4 + 3\sqrt{2}\sin \omega t\right)$, A.



30. Determinate distortion power T of circuit which contain series connected R, L elements if R=4 ohm, $x_L = \omega L$ =3 ohm, and current is $i = \left(4 + 3\sqrt{2}\sin \omega t\right)$, A.



3. THE CALCULATION METHODS OF TRANSIENTS IN LINEAR CIRCUITS

3.1. Study guides as to the calculation of transients in linear circuits

1. Transients calculation by classical approach.
1.1. Transients appear in electric circuits where in reactive elements there are electromagnetic stored energy changes. The stored electromagnetic energy at the finite energy power sources can change only as smooth, without step changes which brings to the transients appear.
1.2. We consider the stored electromagnetic energy change at expense of switchings in the circuit branches. Any switching in circuits we determine by "coomutation" term.
1.3. Switching are performed by means of keys. In electric circuits are used the keys of two types: normally closed contact (normally closed contact, normally-on contact, front-release contact) and normally open contact (make contact, front contact, normally-off contact). The normally closed contact till coomutation have the resistance equal to zero, and after of coomutation this resistance equal to infinity. At normally open contact till coomutation resistance is equal zero, and the after of coomutation is equal to infinity.
1.4. In the circuit transient is described by nonhomogeneous differential equation. The order of differential equation uniquely is determined by quantity of energy storages in electric chain. In the right part of nonhomogeneous differential equation situate the value of quantity which determined by the circuit parameters power supply.

1.5. The nonhomogeneous differential equation solution is defined in the form of the two integrals sum. The first integral is determined by the general solution of homogeneous differential equation and is called this deciding as natural component of transient and second integral is determined by the partial decision of nonhomogeneous differential equation and is called this deciding as forced (enforced) component of transient.
1.6. The transient natural component is defined by the characteristically equation roots. The characteristically equation roots is funded from the operational resistance of electric circuit after commutation. For characteristically equation every root correspond its transient exponential component.
1.7. The real parts of the characteristically equation roots should be negative, that correspond to extinctions transient.
1.8. The value reciprocal to modulus from the characteristically equation root real part is the constant time of transient. The transient to last from three till 5 constants time.
1.9. Deciding of differential equations brings to the necessity of the calculation of integration constant which are calculated based on independent and dependent initial conditions.
1.10. Independent initial conditions are determined on electromagnetic energy storages. Independent initial conditions are calculated based on laws of commutation.
1.11. The first commutation law: in any electric branch comprising inductance element the current and flux linkage at the commutation moment preserve values which they are owned directly before commutation moment, and in further they change beginning from these values. In particular case the first commutation low can be formulated as a commutation rule: current in inductance by stepwise does not change.
1.12. The second commutation law: in any electric branch comprising capacitance element the voltage and charge at the commutation moment preserve values which they are owned directly before commutation moment, and in further they change beginning from these values. In particular case the second commutation low can be formulated as a commutation rule: voltage in capacitance by stepwise does not change.
1.13. The beginning count of transient performed from the commutation moment.
1.14. Dependent initial conditions are calculated based on independent initial conditions and Kirchhoff's laws composed at the commutation moment.

2. The operational method of the transient calculation.
2.1. On the calculation first stage are determined according to the coomutation laws the independent initial condition on storages of electromagnetic energy under the commutation laws.
2.2. On the second stage of calculations pass on from area real variable to operational representations, with this end in view be built the replacement operational scheme, in which nonzero initial conditions on energies storage are taken into account by means of input additional EMFs.

2.3. By the operational replacement scheme we define the sought quantity by means of deciding of algebraic equations by one of known methods: under Kirchhof's laws, of mesh currents, of node potentials, of transforms, of superposition, of equivalent generator.
2.4. On the third calculation stage being looked the value will be obtain in the form of the fractional-rational function as quotient of the polynomials of numerator to the denominator polynomial.
2.5. We pass on from the area of operational images into area real variable. In simplest cases we use the tables of conversions, and generally case by means of the applications of the expansion theorem.

3. The calculation of transient by the variable states method.
3.1. Alongside with branches current and voltages in the capacity of variables there is convenient to choose variables which bring to deciding of differential equations in normal form or Cauchy's form. The normal form of the system of differential equations define that every equation comprises only the first derivation of fitting variable which is written down in the left part of equation. Right part of equation does not comprise derivations and there is linear function of selected state variables and acting in the circuits the energy sources. Such variables are states variable, and the equations are the state equations. Whereat state variables will form the equations system from the minimal number of variables which completely determine transient current and voltage functions in all branches of circuit after commutation.
3.2. The quantity of the first order equations in Cauchy's form in the equations system and variable stats quantity, there is equal to the differential equation order or to energy storages in circuits.
3.2. For electric circuits as state variable conveniently to accept currents in inductive elements $i_L(t)$ and voltages on capacitive elements $u_C(t)$, where there are fulfiled independent initial conditions.
3.3. Using Kirchhof's laws compose the equations system in normal form in which enter the state vector, the energy sources parameters and branches resistance

$$\frac{d}{dt}\overline{X} = A\overline{X} + B\overline{V},$$

where $\overline{X}$ – the state vector; $\overline{V}$ – supply sources vector; A, B – coefficient matrixes, which defined by circuit branch parameters.
3.4. The received differential equations system

$$\frac{d}{dt}\overline{X} = A\overline{X} + B\overline{V}$$

is decided analytical by using the apparatus of matrix transformations or by digital methods by means of differential equations integrating with allowance for initial condition in Cauchy's form.
3.5. Having found the circuit state vector $\overline{X}$, then output vector $\overline{Y}$ is determined as the linear combination of the state vector and the energy sources vectors

$\overline{Y} = C\overline{X} + D\overline{V}$,
where C, D – coefficients matrixes which determined by the circuit branches parameters.

4. Transient calculation on basis of the Duhamel integral (superposition integral)
4.1. By classical or operational methods find transients in circuits at perturbations, when ones has analytic description.
4.2. For the relief of the search the dependent initial condition at the commutation moment in inductive coil suitable to replace ones by ideal current sources, and capacitive elements by the ideal voltage sources. The current value of current source is determined by the first commutation law and the voltage value of the ideal voltage source under the second commutation law. At zero initial conditions can be considered that inductive coil at the commutation moment tears branch, where it is included, and capacitive content shunting the subcircuit branch, where it is included.
4.3. By transient function find normalized transient function upon of single power supply: voltage source with output voltage 1 V or current source with output current 1A. The normalized transient function in depending from dimensions input and output signals can have dimension of resistance, conductions or to be dimensionless value.
4.4. Perform the piecewise-linear approximation of input voltage or current by analytic description by time functions.
4.5. For the separated input curve pieces of piecewise-linear approximation apply one of the forms of Duhamel integral and find the circuit reaction upon given complex input signal as the decisions sum form which are connected on the borders of the approximation pieces.

3.2. The circuit parameters calculation of transients in branched resistive-inductive circuit

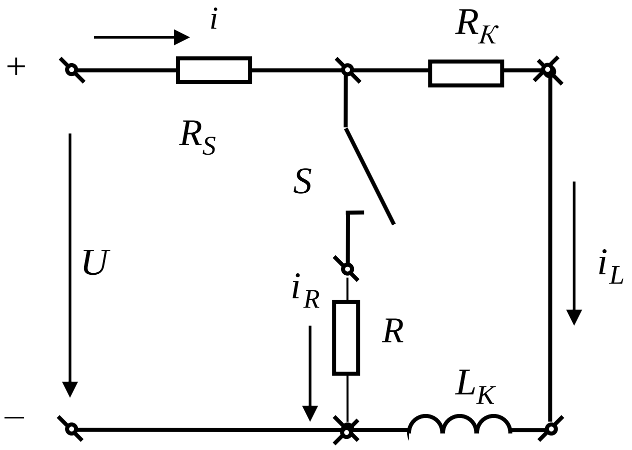


Fig. 3.1

Task.
Calculate transients current and voltage by classical and operational methods in electric circuit, Fig.3.1 that comprising elements: $R_{îãð} = R_K = R = 10$ ohm, $L_K = 0{,}05$ H, $U = 20$ V.

Task solving by classical approach.
Before of commutation moment the circuit comprises only one branch that flowed by current
$i_L(-0) = U/(R_S + R_{Ê}) = 20/20 = 1, À$.

In according to the current's value $i_L(-0)=1,\grave{A}$ before the commutation moment inductive coil stored energy in magnetic field $W_L(-0)=Li^2{}_L(-0)/2$. At termination of transient the inductive coil flowing by direct current (forced response)

$$i_{fL}=\frac{U}{R_S+\dfrac{R_{\hat{E}}\cdot R}{R_{\hat{E}}+R}}\frac{R}{R_{\hat{E}}+R}=0{,}67,\grave{A}.$$

At termination of transient stored electromagnetic energy in inductive coil $W_{fL}(-0)=Li^2{}_{fL}/2$ to change (be decreased), because flowed coil current is reducing. Based on continuity principle of the stored electromagnetic energy in circuit appears transient. For the calculations simplification current and voltages in branches with inductive coil simpler to find current at first, and voltage on inductive coil to calculate as voltage drop from this current.

After commutation moment are generated natural (index *n*) and forced (index *f*) currents in every branches

$$i=i_n+i_f;i_R=i_{Rf}+i_{Rn};i_L=i_{Lf}+i_{Ln}.$$

The current branches forced component find after the transient ending . In the calculation is taken into account, that ideal inductive coil does not accord resistance to flowing direct current, and currents in parallel branches are determined as in divider currents:

$$i_f=\frac{U}{R_S+\dfrac{R_{\hat{E}}\cdot R}{R_{\hat{E}}+R}}=1{,}33,A;i_{Rf}=\frac{U}{R_S+\dfrac{R_{\hat{E}}\cdot R}{R_{\hat{E}}+R}}\frac{R_{\hat{E}}}{R_{\hat{E}}+R}=0{,}67,A;$$

$$i_{Lf}=\frac{U}{R_S+\dfrac{R_{\hat{E}}\cdot R}{R_{\hat{E}}+R}}\frac{R}{R_{\hat{E}}+R}=0{,}67,A.$$

The natural components of transient find as to the roots of characteristically equation. This equation find through circuit resistance for alternating current after commutation moment

$$\underline{Z}=R_S+\frac{(R_K+j\omega L_K)\cdot R}{R_K+j\omega L_K+R}.$$

Into found resistance introduce the characteristically equation root *p* by means of the formal replacement of symbols $j\omega\to p$ and equalization equation to zero

$$R_S+\frac{(R_K+pL_K)\cdot R}{R_K+pL_K+R}=0.$$

We separate the root of characteristically equation

$$R_S+\frac{(R_K+pL_K)\cdot R}{R_K+pL_K+R}=\frac{R_S(R_K+pL_K+R)+(R_K+pL_K)\cdot R}{R_K+pL_K+R}=0.$$

The fractional rational function is equal to zero, when numerator is equal to zero

$R_S(R_K + pL_K + R) + (R_K + pL_K)\cdot R = 0.$

The desired root quantity of characteristically equation

$$p = -\frac{R_S(R_K + R) + R_K \cdot R}{L_K(R_S + R)} = -\frac{10\cdot 20 + 100}{0{,}05\cdot 10^{-3}\cdot 20} = -300000{,}1/\tilde{n}.$$

For found root generally correspond of natural branches current

$i_n = Ae^{pt}; i_{Ln} = A_1e^{pt}; i_{Rn} = A_2e^{pt},$

where A, A_1, A_2 - integrating constants.

For the search of integrating constants find independent initial conditions

$i_L(0) = U/(R_S + R_{\hat{E}}) = 20/20 = 1, \grave{A}.$

Dependent initial conditions find according to independent ones and Kirchhof's laws composed at the commutation moment

$$\left.\begin{aligned} &i_L(0) = 1, \grave{A};\\ &i(0) = i_L(0) + i_R(0);\\ &U = i(0)R_S + i_R(0)R. \end{aligned}\right\}$$

We find solution of the equations system with two unknown currents

$$\left.\begin{aligned} &i(0) = 1 + i_R(0);\\ &20 = 10\cdot i(0) + 10\cdot i_R(0) \end{aligned}\right\} \qquad \left.\begin{aligned} &i_R(0) = 0{,}5, A;\\ &i(0) = 1{,}5, A. \end{aligned}\right\}$$

Having found dependent initial conditions find the integrating constants A, A_1, A_2 by using initial equations transient currents at the commutation moment ($t=0$)

$$\left.\begin{aligned} &i(0) = i_n(0) + i_f(0);\\ &i_R(0) = i_{Rn}(0) + i_{Rf}(0);\\ &i_L(0) = i_{Ln}(0) + i_{Lf}(0); \end{aligned}\right\} \qquad \left.\begin{aligned} &1{,}5 = A + 1{,}33;\\ &0{,}5 = A_2 + 0{,}67;\\ &1 = A_1 + 0{,}67; \end{aligned}\right\} \qquad \left.\begin{aligned} &A = 1{,}5 - 1{,}34 = 0{,}17;\\ &A_2 = 0{,}5 - 0{,}67 = -0{,}17;\\ &A_1 = 1 - 0{,}67 = 0{,}33; \end{aligned}\right\}$$

Found solution for branches current

$$i = 0{,}16e^{-300000\cdot t} + 1{,}33;$$

$$i_R = -0{,}17e^{-300000\cdot t} + 0{,}67;$$

$$i_L = 0{,}33e^{-300000\cdot t} + 0{,}67.$$

The voltage drop on inductive coil

$$u_L = L_K\frac{di_L}{dt} = 0{,}05\cdot 10^{-3}\frac{d(0{,}33e^{-300000\cdot t} + 0{,}67)}{dt} = -5e^{-300000\cdot t}, V.$$

The voltage on inductive coil can be calculated directly as sum of forced and free components

$u_L = u_{Ln} + u_{Lf}.$

The forced component of the voltage drop after ending of transient is equal to zero, because the inductive coil flowing by direct current $u_{Lf} = 0.$

The free component define according to the characteristically equation root $p = -300000, 1/ñ$: $u_{Ln} = A_4 e^{pt}$. We find dependent initial conditions having applicated to the exterior contour second Kirchhof's law

$U = i(0)R_S + i_L(0)R_K + u_L(0);\ 20 = 15 + 10 + u_L(0);\ u_L(0) = -5, V.$

The initial equation at the commutation moment

$u_L(0) = u_{Ln}(0) + u_{Lf}(0);\ -5 = A_4 + 0.$

Desired solution for transient voltage on inductive coil the after of commutation moment coincides with early found value $u_L = -5e^{-300000 \cdot t}, V.$

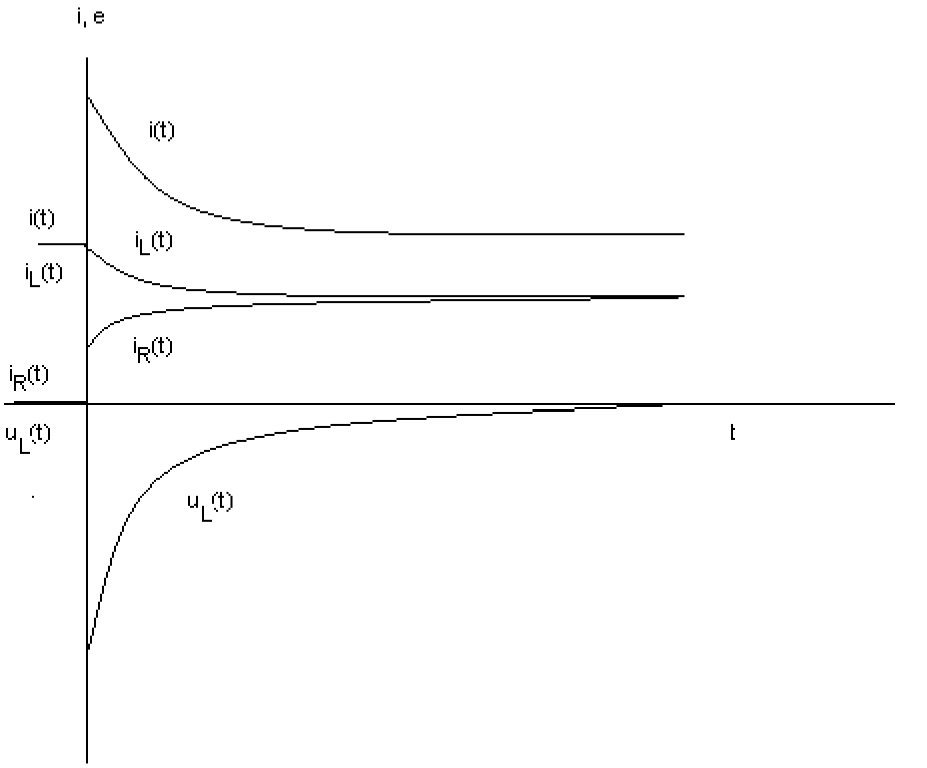


Fig. 3.2.

The graphs of the computative values of the transient voltage on inductive coil and branches currents are presented on Fig. 3.2.

Task solving by operational method.

Nonzero initial conditions on energies storage (coil inductive) defined according to the first commutation law

$i_L(0) = U/(R_S + R_{\hat{E}}) = 20/20 = 1, \grave{A}.$

The operational replacement scheme is built after commutation moment with allowance for nonzero initial conditions, Fig.3.3.

Images of being looked for branch currents find by mesh method in operational form

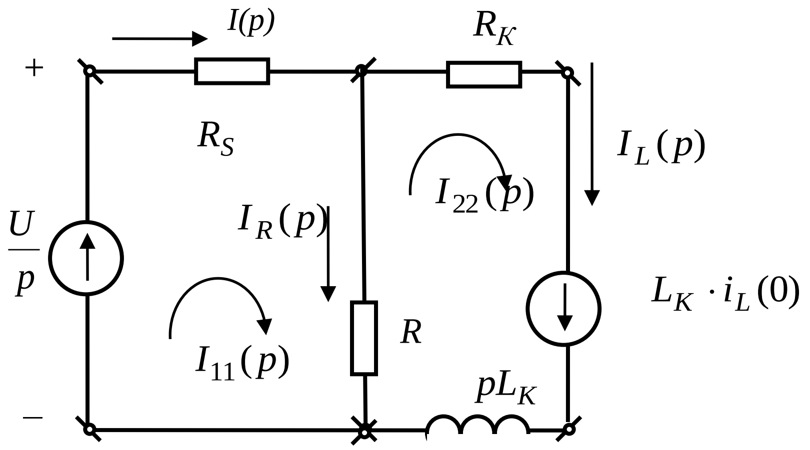


Fig. 3.3

$$\left.\begin{aligned} I_{11}(p)Z_{11}(p) - I_{22}(p)Z_{12}(p) = E_{11}(p) \\ -I_{11}(p)Z_{21}(p) + I_{22}(p)Z_{22}(p) = E_{22}(p) \end{aligned}\right\}$$

$$\left.\begin{aligned} I_{11}(p)(R_S + R) - I_{22}(p)(R) = U/p \\ -I_{11}(p)(R) + I_{22}(p)(R_{\hat{E}} + R + pL_K)Z_{22}(p) = L_K i_L(0) \end{aligned}\right\}$$

$$\left.\begin{aligned} I_{11}(p)(20) - I_{22}(p)(10) = 20/p \\ -I_{11}(p)(10) + I_{22}(p)(20 + p\cdot 0{,}05\cdot 10^{-3}) = 0{,}05\cdot 10^{-3}\cdot 1 \end{aligned}\right\}$$

$$\Delta = \begin{vmatrix} 20 & -10 \\ -10 & 20 + p\cdot 0{,}05\cdot 10^{-3} \end{vmatrix} = 20(20 + p\cdot 0{,}05\cdot 10^{-3}) - 100 = 300 + 0{,}001p;$$

$$\Delta_1 = \begin{vmatrix} 20/p & -10 \\ 0{,}05\cdot 10^{-3} & 20 + p\cdot 0{,}05\cdot 10^{-3} \end{vmatrix} = \frac{20}{p}(20 + p\cdot 0{,}05\cdot 10^{-3}) + 0{,}5\cdot 10^{-3} =$$

$$= \frac{400}{p} + 1{,}5\cdot 10^{-3} = \frac{400 + 1{,}5\cdot 10^{-3}p}{p};$$

$$\Delta_2 = \begin{vmatrix} 20 & 20/p \\ -10 & 0{,}05\cdot 10^{-3} \end{vmatrix} = 1\cdot 10^{-3} + \frac{200}{p} = \frac{1\cdot 10^{-3}p + 200}{p}.$$

The scheme's mesh currents images

$$I_{11}(p) = \frac{\Delta_1}{\Delta} = \frac{\dfrac{400 + 1{,}5\cdot 10^{-3}p}{p}}{300 + 0{,}001p} = \frac{400 + 1{,}5\cdot 10^{-3}p}{p(300 + 0{,}001p)};$$

$$I_{22}(p) = \frac{\Delta_2}{\Delta} = \frac{\dfrac{200 + 1\cdot 10^{-3}p}{p}}{300 + 0{,}001p} = \frac{200 + 1\cdot 10^{-3}p}{p(300 + 0{,}001p)}.$$

The scheme's branch currents images

$$I(p) = I_{11}(p) = \frac{400 + 1{,}5 \cdot 10^{-3} p}{p(300 + 0{,}001p)} = \frac{F_1(p)}{F_2(p)};$$

$$I_R(p) = I_{11}(p) - I_{22}(p) =$$

$$\frac{400 + 1{,}5 \cdot 10^{-3} p}{p(300 + 0{,}001p)} - \frac{200 + 1{,}0 \cdot 10^{-3} p}{p(300 + 0{,}001p)} = \frac{200 + 0{,}5 \cdot 10^{-3} p}{p(300 + 0{,}001p)} = \frac{F_3(p)}{F_4(p)};$$

$$I_L(p) = I_{22}(p) = \frac{200 + 1{,}0 \cdot 10^{-3} p}{p(300 + 0{,}001p)} = \frac{F_5(p)}{F_6(p)}.$$

We find originals of desired currents quantity in functions real variable using the expansion theorem

$$F_1(p) = 400 + 1{,}5 \cdot 10^{-3} p; F_2(p) = F_4(p) = F_6(p) = p(300 + 0{,}001p);$$

$$F_3(p) = 200 + 0{,}5 \cdot 10^{-3} p; F_5(p) = 200 + 1 \cdot 10^{-3} p;$$

$$F_2(p) = F_4(p) = F_6(p) = p(300 + 0{,}001p) = 0 \to p_0 = 0; p_1 = -300000;$$

$$F_2'(p) = F_4'(p) = F_6'(p) = 300 + 0{,}002p.$$

$$i(t) = \sum_{n=0}^{1} \frac{F_1(p_n)}{F_2'(p_n)} e^{p_n t} = \frac{F_1(0)}{F_2'(0)} e^{0t} + \frac{F_1(-300000)}{F_2'(-300000)} e^{-300000t} =$$

$$= \frac{400}{300} + \frac{400 + 1{,}5 \cdot 10^{-3}(-300000)}{300 + 0{,}002 \cdot (-300000)} e^{-300000 \cdot t} = 1{,}33 + 0{,}16 e^{-300000 \cdot t};$$

$$i_R(t) = \sum_{n=0}^{1} \frac{F_3(p_n)}{F_4'(p_n)} e^{p_n t} = \frac{F_3(0)}{F_4'(0)} e^{0t} + \frac{F_3(-300000)}{F_4'(-300000)} e^{-300000t} =$$

$$= \frac{200}{300} + \frac{200 + 0{,}5 \cdot 10^{-3}(-300000)}{300 + 0{,}002 \cdot (-300000)} e^{-300000 \cdot t} = 0{,}67 - 0{,}17 e^{-300000 \cdot t};$$

$$i_L(t) = \sum_{n=0}^{1} \frac{F_5(p_n)}{F_6'(p_n)} e^{p_n t} = \frac{F_5(0)}{F_6'(0)} e^{0t} + \frac{F_5(-300000)}{F_6'(-300000)} e^{-300000t} =$$

$$= \frac{200}{300} + \frac{200 + 1 \cdot 10^{-3}(-300000)}{300 + 0{,}002 \cdot (-300000)} e^{-300000 \cdot t} = 0{,}67 + 0{,}33 e^{-300000 \cdot t}.$$

The operational image of the voltage drop on inductive coil find under the law Ohm's in operational form

$$U_L(p) = I_L(p) p L_K = \frac{200 + 1{,}0 \cdot 10^{-3} p}{p(300 + 0{,}001p)} p \cdot 0{,}05 \cdot 10^{-3} =$$

$$= \frac{0{,}01p + 0{,}05 \cdot 10^{-6} p}{p(300 + 0{,}001p)} = \frac{F_7(p)}{F_8(p)}.$$

The voltage original on inductive coil find having applicated the expansion theorem

$$u_L(t) = \sum_{n=0}^{1} \frac{F_7(p_n)}{F_8'(p_n)} e^{p_n t} = \frac{F_7(0)}{F_8'(0)} e^{0t} + \frac{F_7(-300000)}{F_8'(-300000)} e^{-300000t} =$$

$$= \frac{0}{300} + \frac{0{,}01(-300000) + 0{,}05 \cdot 10^{-6}(-300000)}{300 + 0{,}002 \cdot (-300000)} e^{-300000 \cdot t} = -5e^{-300000 \cdot t}, V;$$

3.3. The circuit parameters calculation of transients in branched resistive-capacitive circuit

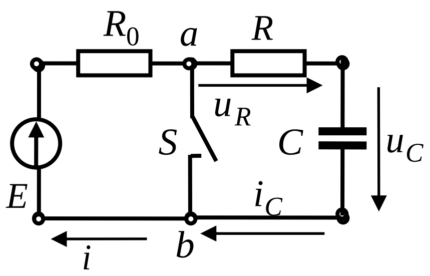


Fig.3.4

Task.
Calculate of transient currents and voltages by classic and operational methods in electric circuit, Fig.3.4 which comprising elements: $R_O = R = 10$ ohm, $C = 10\ \mu F$, $E = 20$ V.

Task solving by classical approach.
In branches comprising capacitive elements more convenient in the beginning to find voltage on capacitances, and current to determine as derivative from the calculated capacitive voltage. Branch resistance comprising key S after of commutation moment is equal to zero, that is why the scheme after of commutation moment breaks into two independent contour: left and right ones, in which flowing independent transients.

We reckon of branches current and voltage on capacitance by the superposition method of forced and natural components

$i = i_n + i_f; i_C = i_{Cn} + i_{Cf}; u_C = i_{Cn} + i_{Cf}$.

The forced component find in circuit at transient termination. We take into account that ideal the capacitive element direct current does not conducting

$$i_f = \frac{E}{R_0} = 2{,}0, A; i_{Cf} = 0, A; u_{Cf} = 0, V.$$

General solutions for transient natural component determine according to the characteristically equation roots. Roots find as to input resistance for alternating current. Because characteristically equation is common for all circuit therefore in this calculation more convenient to find the input resistance relatively of branch with capacitive

$$\underline{Z} = \frac{1}{j\omega C} + R.$$

Work into last equation the root of characteristically equation by means of the formal symbol replacement $j\omega$ on symbol p and after equalization the equation to zero, shall receive

$$\frac{1}{pC}+R=0.$$

From where the value of the characteristically equation root

$$p=-\frac{1}{RC}=-\frac{1}{10\cdot 10\cdot 10^{-6}}=1\cdot 10^{-4}, c.$$

The time constant of transient $\tau=\left|\frac{1}{p}\right|=\frac{1}{10000}=10000{,}1/c.$

For the characteristically equation root correspond currents and voltage natural component

$$i_n=Ae^{pt}; i_{Cn}=A_1e^{pt}; u_{Cn}=A_2e^{pt},$$

where A, A_1, A_2 - integrating constants.

For the integrating constant define we find independent initial conditions. As to the second commutation law the voltage on capacitive element

$$u_C(0)=E=20, V.$$

Dependent initial condition find as to independent ones and Kirchhof's laws which composed at the commutation moment

$$\left.\begin{array}{l} E=i(0)\cdot R_0; \\ i_C(0)\cdot R+u_C(0)=0. \end{array}\right\} \quad \left.\begin{array}{l} i(0)=E/R_0=2; \\ i_C(0)=-u_C(0)/R=-2. \end{array}\right\}$$

Having found dependent initial conditions find the integrating constants A, A_1, A_2 using initial equations currents transient at the commutation moment in *t=0*

$$i(0)=i_{\tilde{n}\hat{a}}(0)+i_{\ddot{i}\eth}(0); i_C(0)=i_{C\tilde{n}\hat{a}}(0)+i_{C\ddot{i}\eth}(0); u_C(0)=i_{C\tilde{n}\hat{a}}(0)+i_{C\ddot{i}\eth}(0).$$

Subdtituting the digital values obtain

$$2{,}0=A+2{,}0;$$

$$-2{,}0=A_2+0{,}0;$$

$$20=A_1+0{,}0;$$

$$A=0;$$

$$A_2=-2;$$

$$A_1=20.$$

Found deciding for current branches and voltage on capacitance

$$i=2{,}0;$$

$$i_C=-2{,}0e^{-10000\cdot t};$$

$$u_C=20e^{-10000\cdot t}.$$

Absence natural current component in branch with power supply bears evidence about step change current from initial zero value till two amper without the

generation natural component of transient. This is explained by storage energies absence in contour with power supply after commutation.
The capacitive current is defined through derivative from capacitance voltage

$$i_C = C\frac{du_C}{dt} = 10\cdot 10^{-6}\frac{d(20e^{-10000\cdot t})}{dt} = -2{,}0e^{-10000\cdot t},$$

There is directly confirming the solution as correct.

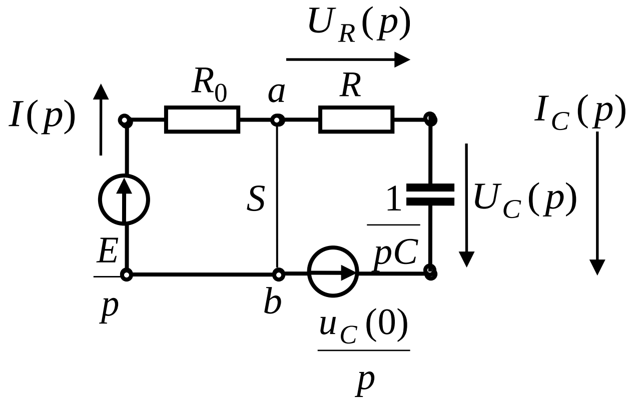


Fig.3.5

Task solving by operational method.
The nonzero initial condition on energy storage (capacitive element) define as to the second commutation law $u_C(0) = E = 20, V$.
The operational replacement scheme is built after commutation moment with allowance for nonzero initial conditions, Fig.3.5.
The operational image of desired branches current find under the Ohm's law in operational form, because transients in contours are independent

$$I(p) = \frac{E(p)}{R_0} = \frac{E}{pR_0} = \frac{20}{10p} = \frac{F_1(p)}{F_2(p)},$$

$$I_C(p) = -\frac{u_C(0)/p}{\frac{1}{pC}+R} = -\frac{u_C(0)\cdot C}{1+pRC} = -\frac{20\cdot 10\cdot 10^{-6}}{1+p\cdot 10\cdot 10\cdot 10^{-6}} = \frac{F_3(p)}{F_4(p)},$$

We find originals desired currents in functions real variable using the expansion theorem

$F_1(p) = 20; F_2(p) = pR_0; F_2(p) = pR_0 = 0 \to p_0 = 0;$

$F_2'(p) = R_0 = 10;$

$F_3(p) = -200\cdot 10^{-6}; F_4(p) = 1 + 100\cdot 10^{-6}p; F_4'(p) = 100\cdot 10^{-6};$

$F_4(p) = 1 + 100\cdot 10^{-6}p = 0 \to p_1 = -10000.$

$$i(t) = \sum_{n=0}\frac{F_1(p_0)}{F_2'(p_0)}e^{p_0 t} = \frac{F_1(0)}{F_2'(0)}e^{0t} = \frac{20}{10} = 2{,}0A;$$

$$i_C(t)=\sum_{n=1}\frac{F_3(p_n)}{F_4'(p_n)}e^{p_n t}=\frac{F_3(-10000)}{F_4'(-10000)}e^{-10000t}=$$

$$=\frac{-200\cdot 10^{-6}}{100\cdot 10^{-6}}e^{-10000\cdot t}=-2{,}0e^{-10000\cdot t}, A.$$

We shall find the operational image voltage drop on capacitance under the Ohm's law

$$U_C(p)=I_C(p)\cdot\frac{1}{pC}=-\frac{u_C(0)\cdot C}{1+pRC}\cdot\frac{1}{pC}=-\frac{u_C(0)}{1+pRC}\cdot\frac{1}{p}=\frac{F_5(p)}{F_6(p)};$$

The original of desired voltage on capacitive in real variable function we find by using the expansion theorem

$$F_5(p)=-u_C(0)=-20; F_6(p)=pF_4(p)=p(1+RCp);$$

$$F_6(p)=p(1+0{,}0001p)=0\rightarrow p_0=0;\ p_1=-10000;$$

$$F_6'(p)=1+2RCp=1+0{,}0002p.$$

$$i(t)=\sum_{n=0}^{1}\frac{F_5(p_n)}{F_6'(p_n)}e^{p_n t}=\frac{F_5(0)}{F_6'(0)}e^{0t}+\frac{F_5(-10000)}{F_6'(-10000)}e^{-10000t}=$$

$$=\frac{-20}{1}+\frac{-20}{1+0{,}0002\cdot(-10000)}e^{-10000\cdot t}=-20+20e^{-10000\cdot t}, V.$$

The calculation result of the voltage drop by ope3rational method reveals that in final equation is taken into account not only voltage on capacitive element but also nonzero initial condition on the capacitor.

3.4. The circuit parameters calculation of transients in branched resistive-inductive-capacitive circuit

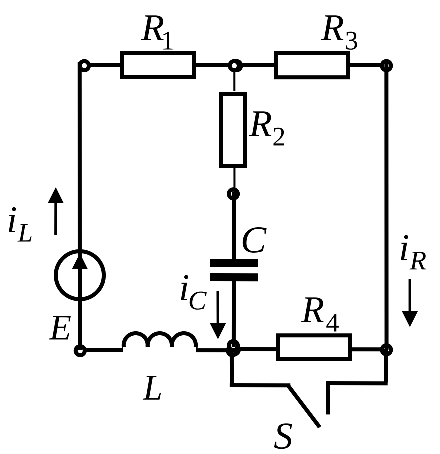


Fig. 3.6

Task.
Calculate of transient currents and voltage by classic and operational methods in electric circuit, Fig.3.6, which comprising elements $R_1=0{,}1$ ohm; $R_2=0{,}1$ ohm; $R_3=10$ ohm, $R_4=10$ ohm, $L=1{,}0$ mH, $C=30$ μF , $E=20$ V.

Task solving by classical approach.
In order to avoid the operations of integrating of eventual results, find branches current and voltage on capacitor. The drop voltage on inductive element we shall find as derivative from current in inductive one. We present transients as sum enforced and natural components

$$i_L=i_{Ln}+i_{Lf}; i_C=i_{Cn}+i_{Cf}; i_R=i_{Rn}+i_{Rf}; u_C=i_{Cn}+i_{Cf}.$$

Transient functions forced component find in chains at termination transient, the taking into account null resistance of ideal inductive coil and the resistance infinitely large of capacitor to direct current

$$i_{Lf} = \frac{E}{R_1 + R_3} = \frac{20}{0{,}1 + 10} = 1{,}98, A;$$

$$i_{Cf} = 0;$$

$$i_{Rf} = i_{Lf} = 1{,}98, A;$$

$$u_{Cf} = i_{Rf} \cdot R_3 = 1{,}98 \cdot 10 = 19{,}8, V.$$

For determining transient natural component compose characteristically equation using the circuitt resistance to alternating current. From circuit delete energy sources leaving their inner resistance into calculated scheme, and tearing branch with capacitor, we have

$$\underline{Z} = \frac{1}{j\omega C} + R_2 + \frac{(R_1 + j\omega L)R_3}{R_3 + R_1 + j\omega L}.$$

By introducing into received equation the characteristically equation root by means of formal replacement the symbol of p instead symbol $j\omega$ and received equation set equal to zero, we shall receive

$$\frac{1}{pC} + R_2 + \frac{(R_1 + pL)R_3}{R_3 + R_1 + pL} = 0;$$

$$\frac{(R_3 + R_1 + pL) + R_2 pC(R_3 + R_1 + pL) + pC(R_1 + pL)R_3}{pC(R_3 + R_1 + pL)} = 0.$$

The fraction is equal to zero, when numerator is equal to zero

$$(R_3 + R_1 + pL) + R_2 pC(R_3 + R_1 + pL) + pC(R_1 + pL)R_3 = 0.$$

We do ordering of the polynomial relatively of characteristically equation root p

$$p^2 CL(R_2 + R_3) + p(L + C(R_3R_2 + R_1R_2 + R_1R_3)) + (R_3 + R_1) = 0.$$

$$p^2 3{,}03 \cdot 10^{-7} + p \cdot 1{,}06 \cdot 10^{-3} + 10{,}1 = 0.$$

We define the roots of characteristically equation

$$p_{1,2} = \frac{-1{,}06 \cdot 10^{-3} \pm \sqrt{(1{,}06 \cdot 10^{-3})^2 - 4 \cdot 3{,}03 \cdot 10^{-7} \cdot 10{,}1}}{2 \cdot 3{,}03 \cdot 10^{-7}} =$$

$$= -1749 \pm j5502 = \delta \pm j\omega.$$

Received roots are conjugated that bears evidence about oscillating transient with attenuation decrement $\delta = -99{,}75$ and by the frequency of free oscillating $\omega = 5502 rad/s$. The transient constant time $\tau = \left|\delta^{-1}\right| = 1749^{-1} = 5{,}7 \cdot 10^{-4}, s.$

For found roots correspond generally form the nature compoonents of transeint functions

$$i_{Ln} = B_1 e^{\delta t}\sin(\omega t + \beta_1);$$
$$i_{Cn} = B_2 e^{\delta t}\sin(\omega t + \beta_2);$$
$$i_{Rn} = B_3 e^{\delta t}\sin(\omega t + \beta_3);$$
$$u_{Cn} = B_4 e^{\delta t}\sin(\omega t + \beta_4),$$
where $B_1, B_2, B_3, \beta_1, \beta_2, \beta_3$ - unknown integration constants.

Independent initial condition reckon under the commutation laws

$$i_L(0) = \frac{E}{R_1 + R_3 + R_4} = \frac{20}{0{,}1 + 10 + 10} = 0{,}995, A;$$

$$u_C(0) = i_L(0)(R_3 + R_4) = 0{,}995 \cdot 20 = 19{,}9, V.$$

Dependent initial conditions find as to independent ones and Kirchhof's laws which compose at the commutation moment (*t=0*)

$$\left.\begin{array}{l} i_L(0) = 0{,}995\text{À}; \\ u_C(0) = 19{,}9, V; \\ i_L(0) = i_C(0) + i_R(0); \\ i_C(0)R_2 + u_C(0) = i_R(0)R_3; \\ E = i_L(0)R_1 + i_R(0)R_3 + L\left.\dfrac{di_L}{dt}\right|_{t=0} \end{array}\right\} \quad \left.\begin{array}{l} 0{,}995 = i_C(0) + i_R(0); \\ 0{,}1 \cdot i_C(0) + 19{,}9 = 10 \cdot i_R(0); \\ 20 = 0{,}0995 + 10 \cdot i_R(0) + 1 \cdot 10^{-3}\left.\dfrac{di_L}{dt}\right|_{t=0} \end{array}\right\}$$

$$\left.\begin{array}{l} i_L(0) = 0{,}995; \\ i_C(0) = -0{,}985; \\ i_R(0) = 1{,}98; \\ \left.\dfrac{di_L}{dt}\right|_{t=0} = 100{,}5, A/s. \end{array}\right\}$$

We find dependent initial conditions for current in capacitance and omhic resistance by means of differentiation the initial equations system

$$\left.\begin{array}{l} i_L(0) = i_C(0) + i_R(0); \\ i_C(0)R_2 + u_C(0) = i_R(0)R_3; \end{array}\right\}$$

over the time

$$\left.\begin{array}{l} \left.\dfrac{di_L}{dt}\right|_{t=0} = \left.\dfrac{di_C}{dt}\right|_{t=0} + \left.\dfrac{di_R}{dt}\right|_{t=0}; \\ R_2\left.\dfrac{di_C}{dt}\right|_{t=0} + \dfrac{i_C(0)}{C} = R_3\left.\dfrac{di_R}{dt}\right|_{t=0}; \end{array}\right\} \quad \left.\begin{array}{l} 0{,}5 = \left.\dfrac{di_C}{dt}\right|_{t=0} + \left.\dfrac{di_R}{dt}\right|_{t=0}; \\ 0{,}1 \cdot \left.\dfrac{di_C}{dt}\right|_{t=0} + \dfrac{-0{,}985}{30 \cdot 10^{-6}} = 10 \cdot \left.\dfrac{di_R}{dt}\right|_{t=0}; \end{array}\right\}$$

$$\left.\frac{di_C}{dt}\right|_{t=0} = 3251{,}32;$$

$$\left.\frac{di_R}{dt}\right|_{t=0} = -3250{,}82.$$

where is taken into account $u_C = \frac{1}{C}\int i_C dt \rightarrow \frac{du_C}{dt} = \frac{i_C}{C} \rightarrow \left.\frac{du_C}{dt}\right|_{t=0} = \frac{i_C(0)}{C}.$

Having found dependent initial conditions find the integrating constant $B_1, B_2, B_3, \beta_1, \beta_2, \beta_3$ in consideration of initial transient currents and voltage

$$\left.\begin{aligned}
i_L &= i_{Lf} + i_{Ln} = i_{Lf} + B_1 e^{\delta t}\sin(\omega t + \beta_1);\\
i_C &= i_{Cf} + i_{Cn} = i_{Cf} + B_2 e^{\delta t}\sin(\omega t + \beta_2);\\
i_R &= i_{Rf} + i_{Rn} = i_{Rf} + B_3 e^{\delta t}\sin(\omega t + \beta_3);\\
u_C &= u_{Cf} + u_{Cn} = u_{Cf} + B_4 e^{\delta t}\sin(\omega t + \beta_4).
\end{aligned}\right\}$$

In the last equations system in every equation there is two unknown B, β. So as to the number of equations corresponded number of unknowns, from last equations find derivative over the time

$$\left.\begin{aligned}
\frac{di_L}{dt} &= B_1\delta \cdot e^{\delta t}\sin(\omega t + \beta_1) + B_1 e^{\delta t}\omega \cdot \cos(\omega t + \beta_1);\\
\frac{di_C}{dt} &= B_2\delta \cdot e^{\delta t}\sin(\omega t + \beta_2) + B_2 e^{\delta t}\omega \cdot \cos(\omega t + \beta_2);\\
\frac{di_R}{dt} &= B_3\delta \cdot e^{\delta t}\sin(\omega t + \beta_3) + B_3 e^{\delta t}\omega \cdot \cos(\omega t + \beta_3);\\
\frac{du_C}{dt} &= B_4\delta \cdot e^{\delta t}\sin(\omega t + \beta_4) + B_4 e^{\delta t}\omega \cdot \cos(\omega t + \beta_4);
\end{aligned}\right\}$$

and after consideration of the two last systems of equations at the moment of commutation (*t=0*)

$$\left.\begin{aligned}
i_L(0) &= i_{Lf}(0) + i_{Ln}(0) = 1{,}98 + B_1\sin(\beta_1);\\
i_C(0) &= i_{Cf}(0) + i_{Cn}(0) = 0 + B_2\sin(\beta_2);\\
i_R(0) &= i_{Rf}(0) + i_{Rn}(0) = 1{,}98 + B_3\sin(\beta_3);\\
u_C(0) &= u_{Cf}(0) + u_{Cn}(0) = 19{,}8 + B_4\sin(\beta_4).
\end{aligned}\right\}$$

$$\left.\begin{aligned}
&\left.\frac{di_L}{dt}\right|_{t=0} = B_1\delta\cdot\sin(\beta_1) + B_1\omega\cdot\cos(\beta_1);\\
&\left.\frac{di_C}{dt}\right|_{t=0} = B_2\delta\cdot\sin(\beta_2) + B_2\omega\cdot\cos(\omega\beta_2);\\
&\left.\frac{di_R}{dt}\right|_{t=0} = B_3\delta\cdot\sin(\beta_3) + B_3\omega\cdot\cos(\beta_3);\\
&\left.\frac{du_C}{dt}\right|_{t=0} = B_4\delta\cdot\sin(\beta_4) + B_4\omega\cdot\cos(\beta_4).
\end{aligned}\right\}$$

We substitute digital values into the system of equations

$$\left.\begin{aligned}
&0{,}995 = i_{Lf}(0) + i_{Ln}(0) = 1{,}98 + B_1\sin(\beta_1);\\
&-0{,}985 = i_{Cf}(0) + i_{Cn}(0) = 0 + B_2\sin(\beta_2);\\
&1{,}90 = i_{Rf}(0) + i_{Rn}(0) = 1{,}98 + B_3\sin(\beta_3);\\
&19{,}9 = u_{Cf}(0) + u_{Cn}(0) = 19{,}8 + B_4\sin(\beta_4),\\
&100{,}5 = B_1\delta\cdot\sin(\beta_1) + B_1\omega\cdot\cos(\beta_1);\\
&3251{,}32 = B_2\delta\cdot\sin(\beta_2) + B_2\omega\cdot\cos(\omega\beta_2);\\
&-3250{,}82 = B_3\delta\cdot\sin(\beta_3) + B_3\omega\cdot\cos(\beta_3);\\
&\frac{i_C(0)}{C} = \frac{-0{,}985}{30\cdot10^{-6}} = -32833{,}33 = B_4\delta\cdot\sin(\beta_4) + B_4\omega\cdot\cos(\beta_4);
\end{aligned}\right\}$$

For example we find B_3, β_3 for current i_R. With this end in view from the last equations system choose two equations comprising two unknown values B_3, β_3

$$\left.\begin{aligned}
&1{,}90 = i_{Rf}(0) + i_{R\tilde{n}\hat{a}}(0) = 1{,}98 + B_3\sin(\beta_3);\\
&-3250{,}82 = B_3\delta\cdot\sin(\beta_3) + B_3\omega\cdot\cos(\beta_3);
\end{aligned}\right\}$$

$$\left.\begin{aligned}
&B_3\sin(\beta_3) = -0{,}08;\\
&-3250{,}82 = B_3\delta\cdot\sin(\beta_3) + B_3\omega\cdot\cos(\beta_3);
\end{aligned}\right\}$$

$$-3250{,}82 = B_3\delta\cdot(-0{,}08) + B_3\omega\cdot\cos(\beta_3);$$

$$-3250{,}82 = (-1749)\cdot(-0{,}08) + 5502B_3\cdot\cos(\beta_3);$$

$$B_3\cdot\cos(\beta_3) = -0{,}616; B_3\sin(\beta_3) = -0{,}08;$$

$$\frac{B_3\sin(\beta_3)}{B_3\cdot\cos(\beta_3)} = tg(\beta_3) = \frac{-0{,}08}{-0{,}616} = 0{,}129; \beta_3 = arctg(0{,}129) = 7{,}39^0;$$

$$B_3 = \frac{-0{,}08}{\sin(\beta_3)} = \frac{-0{,}08}{\sin(7{,}39^0)} = -0{,}62, A;$$

Deciding for transient current i_R have of the form

$$i_R = i_{Rf} + i_{Rn} = i_{Rf} + B_3e^{\delta\cdot t}\sin(\omega t + \beta_3) =$$

$$= 1{,}98 - 0{,}62e^{-1749\cdot t}\sin(5502t + 7{,}39^0), A.$$

The integrating constants rest B, β define similarly from the same system of equations by means of the choice the equations pairs comprising with two unknown values B, β

$$\left.\begin{aligned} i_L &= i_{Lf} + i_{Ln} = 1{,}98 + 0{,}985e^{-99{,}75\cdot t}\sin(5772{,}64t - 89{,}9^0), A;\\ i_C &= i_{Cf} + i_{Cn} = 1{,}126\cdot e^{-99{,}75\cdot t}\sin(5772{,}64t - 60{,}9^0), A;\\ i_R &= i_{Rf} + i_{Rn} = 1{,}98 - 0{,}56e^{-99{,}74\cdot t}\sin(5772{,}64t + 8{,}13^0);\\ u_C &= u_{Cf} + u_{Cn} = 19{,}8 - 5{,}72e^{\delta t}\sin(\omega t - 1^0), V. \end{aligned}\right\}$$

Task solving by operational method.

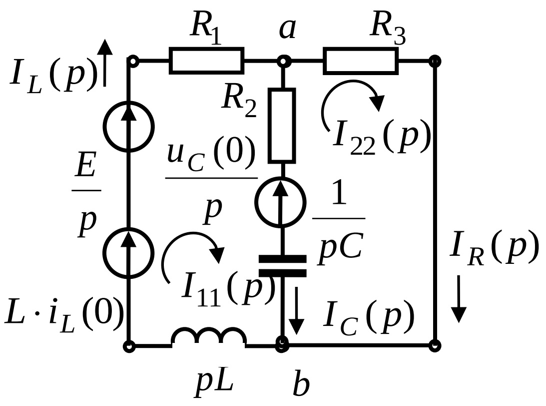


Fig. 3.7

Unnull initial conditions on storage energies (capacitive and inductive elements) define under the commutation laws

$$i_L(0) = \frac{E}{R_1 + R_3 + R_4} = \frac{20}{0{,}1 + 10 + 10} = 0{,}995, A;$$

$$u_C(0) = i_L(0)(R_3 + R_4) = 0{,}995 \cdot 20 = 19{,}9, V.$$

The operation replacement scheme is built after commutation moment with allowance for unnull initial conditions, Fig.3.7.

The branches currents calculation we perform in operational form by method of mesh current

$$\left.\begin{aligned} &Z_{11}(p)I_{11}(p) - Z_{12}I_{22}(p) = E_{11}(p);\\ &-Z_{21}(p)I_{22}(p) + Z_{22}(p)I_{22}(p) = E_{22}(p), \end{aligned}\right\}$$

$$\left.\begin{aligned} &(R_1 + R_2 + \frac{1}{pC} + pL)I_{11}(p) - (R_2 + \frac{1}{pC})I_{22}(p) = Li_L(0) - \frac{u_C(0)}{p} + \frac{E}{p};\\ &-(R_2 + \frac{1}{pC})I_{22}(p) + (R_3 + R_2 + \frac{1}{pC})I_{22}(p) = E_{22}(p), \end{aligned}\right\}$$

$$\left.\begin{aligned} &\frac{\left(pL + R_1 + R_2\right)pC + 1}{pC}I_{11}(p) - \frac{pCR_2 + 1}{pC}I_{22}(p) = \frac{Li_L(0)p - u_C(0) + E}{p};\\ &-\frac{pCR_2 + 1}{pC}I_{22}(p) + \frac{\left(R_3 + R_2\right)pC + 1}{pC}I_{22}(p) = \frac{u_C(0)}{p}, \end{aligned}\right\}$$

$$\Delta=\begin{vmatrix}\frac{(pL+R_1+R_2)pC+1}{pC} & -\frac{pCR_2+1}{pC}\\ -\frac{pCR_2+1}{pC} & \frac{(R_3+R_2)pC+1}{pC}\end{vmatrix}=$$

$$=\frac{p^2LC(R_3+R_2)+p(L+C(R_1+R_2)(R_3+R_2)-R_2^2)+((R_1+R_2)+(R_3+R_2)-2R_2)}{pC}.$$

$$p^2CL(R_2+R_3)+p(L+C(R_3R_2+R_1R_2+R_1R_3))+(R_3+R_1)=0.$$

$$\Delta_1=\begin{vmatrix}\frac{Li_L(0)p-u_C(0)+E}{p} & -\frac{pCR_2+1}{pC}\\ \frac{u_C(0)}{p} & \frac{(R_3+R_2)pC+1}{pC}\end{vmatrix}=$$

$$=\frac{p^2(Li_L(0)(R_3+R_2)C)+p(Li_L(0)+(E-u_C(0))(R_3+R_2)C+u_C(0)CR_2)+E}{p^2C};$$

$$\Delta_2=\begin{vmatrix}\frac{(pL+R_1+R_2)pC+1}{pC} & \frac{Li_L(0)p-u_C(0)+E}{p}\\ -\frac{pCR_2+1}{pC} & \frac{u_C(0)}{p}\end{vmatrix}=$$

$$=\frac{p^2(CLu_C(0)+Li_L(0)CR_2)+p((R_1+R_2)Cu_C(0)+Li_L(0)+CR_2(E-u_C(0)))+E}{p^2C};$$

$$I_{11}(p)=\frac{\Delta_1}{\Delta}=\frac{\dfrac{\begin{array}{c}p^2(Li_L(0)(R_3+R_2)C)+p(Li_L(0)+\\+(E-u_C(0))(R_3+R_2)C+u_C(0)CR_2)+E\end{array}}{p^2C}}{\dfrac{\begin{array}{c}p^2LC(R_3+R_2)+p(L+C(R_1+R_2)(R_3+R_2)-R_2^2)+\\+((R_1+R_2)+(R_3+R_2)-2R_2)\end{array}}{pC}}=$$

$$=\frac{p^2(Li_L(0)(R_3+R_2)C)+p(Li_L(0)+(E-u_C(0))(R_3+R_2)C+u_C(0)CR_2)+E}{p\left(\begin{array}{c}p^2LC(R_3+R_2)+p(L+C(R_1+R_2)(R_3+R_2)-\\-R_2^2)+((R_1+R_2)+(R_3+R_2)-2R_2)\end{array}\right)};$$

$$I_{22}(p)=\frac{\Delta_2}{\Delta}=$$

$$=\frac{p^2(CLu_C(0)+Li_L(0)CR_2)+p((R_1+R_2)Cu_C(0)+Li_L(0)+CR_2(E-u_C(0)))+E}{p\left(\begin{array}{c}p^2LC(R_3+R_2)+p(L+C(R_1+R_2)(R_3+R_2)-R_2^2)+\\+((R_1+R_2)+(R_3+R_2)-2R_2)\end{array}\right)};$$

Operation image of the scheme branches current

$$I_L(p)=I_{11}(p)=$$

$$=\frac{p^2(Li_L(0)(R_3+R_2)C)+p(Li_L(0)+(E-u_C(0))(R_3+R_2)C+u_C(0)CR_2)+E}{p\left(p^2LC(R_3+R_2)+p(L+C(R_1+R_2)(R_3+R_2)-R_2^2)+((R_1+R_2)+(R_3+R_2)-2R_2)\right)}=$$

$$=\frac{a_{12}p^2+a_{11}p+a_{10}}{(b_{12}p^2+b_{11}p+b_{10})p}=\frac{N_1(p)}{D_1(p)};$$

$$I_R(p)=I_{22}(p)=$$

$$=\frac{p^2(CLu_C(0)+Li_L(0)CR_2)+p((R_1+R_2)Cu_C(0)+Li_L(0)+CR_2(E-u_C(0)))+E}{p\begin{pmatrix}p^2LC(R_3+R_2)+p(L+C(R_1+R_2)(R_3+R_2)-R_2^2)+\\+((R_1+R_2)+(R_3+R_2)-2R_2)\end{pmatrix}}=$$

$$=\frac{a_{22}p^2+a_{21}p+a_{20}}{(b_{22}p^2+b_{21}p+b_{20})p}=\frac{N_2(p)}{D_2(p)};$$

$$I_C(p)=I_{11}(p)-I_{22}(p)=$$

$$=\frac{((E-u_C(0))(R_3)C-(R_1)Cu_C(0))}{p^2LC(R_3+R_2)+p(L+C(R_1+R_2)(R_3+R_2)-R_2^2)+((R_1+R_2)+(R_3+R_2)-2R_2)}$$

$$=\frac{a_{30}}{b_{32}p^2+b_{31}p+b_{30}}=\frac{N_3(p)}{D_3(p)};$$

We find current original i_R having applicated to it operational image the expansion theorem

$$i_R=\frac{N_2(p_0)}{D_2'(p_0)}e^{p_0t}+2\operatorname{Re}\frac{N_2(p_1)}{D_2'(p_1)}e^{p_1t}=\frac{a_{20}}{b_{20}}+2\operatorname{Re}\frac{2a_{22}p_1+a_{21}}{3b_{22}p_1^{\ 2}+2b_{21}p_1+b_{20}}=\frac{E}{R_1+R_3}+$$

$$+2\operatorname{Re}\frac{\begin{array}{l}p_1^{\ 2}(CLu_C(0)+Li_L(0)CR_2)+p_1((R_1+R_2)Cu_C(0)+\\+Li_L(0)+CR_2(E-u_C(0)))+E\end{array}}{\begin{array}{l}3p_1^{\ 2}LC(R_3+R_2)+p_1(L+C(R_1+R_2)(R_3+R_2)-R_2^2)+\\+((R_1+R_2)+(R_3+R_2)-2R_2)\end{array}}.$$

We find the roots of polynomial

$$D_2(p)=p^2LC(R_3+R_2)+p(L+C(R_1+R_2)(R_3+R_2)-R_2^2)+$$
$$+((R_1+R_2)+(R_3+R_2)-2R_2)$$

$$p_0=0;\ p_{1,2}=-1749\pm j5502.$$

After substitute the roots value and the accentuations of real part we shall receive

$$i_R=1{,}98-0{,}62e^{-1749\cdot t}\sin(5502t+7{,}39^0),A.$$

Task solving by variable states method.

As the component of the vector of variable states choose current in inductance and voltage on capacitance. Relatively chosen variable states under second Kirchhgof's law compose the equations of relatively independent contours

$$\left.\begin{aligned} &E = i_L R_1 + CR_2 \frac{du_C}{dt} + u_C + L\frac{di_L}{dt}; \\ &CR_2 \frac{du_C}{dt} + u_C = \left(i_L - C\frac{du_C}{dt} \right) R_3; \end{aligned}\right\}$$

From the equations system we separate out explicitly equations in normal Cauchy's form

$$\left.\begin{aligned} &\frac{du_C}{dt} = \frac{i_L R_3 - u_L}{C(R_2 + R_3)}; \\ &\frac{di_L}{dt} = \frac{E}{L} - \frac{u_L}{L} - \frac{R_1}{L} i_L - \frac{(i_L R_3 - u_C) R_2}{L(R_2 + R_3)}; \end{aligned}\right\}$$

From differential equations we pass on to incremental equations

$$\left.\begin{aligned} &\frac{\Delta u_C}{\Delta t} = \frac{i_L R_3 - u_L}{C(R_2 + R_3)}; \\ &\frac{\Delta i_L}{\Delta t} = \frac{E}{L} - \frac{u_L}{L} - \frac{R_1}{L} i_L - \frac{(i_L R_3 - u_C) R_2}{L(R_2 + R_3)}; \end{aligned}\right\}$$

By the first order Euler,s method we shall integrate the received equations system (*κ* – the integrating running interval of ; *κ-1* – the previous interval of integrating; Δt – integrating step).

$$\left.\begin{aligned} &\Delta u_{Ck} = u_{Ck} - u_{Ck-1} = \left(\frac{i_{Lk-1} R_3 - u_{Lk-1}}{C(R_2 + R_3)} \right) \Delta t; \\ &\Delta i_L = i_{Lk} - i_{Lk-1} = \left(\frac{E}{L} - \frac{u_{Lk-1}}{L} - \frac{R_1}{L} i_{Lk-1} - \frac{(i_{Lk-1} R_3 - u_{Ck-1}) R_2}{L(R_2 + R_3)} \right) \Delta t; \end{aligned}\right\}$$

The more exact integrating method there is Runge-Kutt method, which actualized in environment of MathCAD. The printout of the algorithm of the calculation of transient by the variable states method in environment of MathCAD, is present on Fig. 3.8.

The calculation of transient by the variable states method in linear circuit with two energy storages

The circuit parameters initial value assignment (Fig. 15)

$$E := 20 \quad L := 10^{-3} \quad C := 3\times 10^{-5} \quad R1 := 0.1 \quad R3 := 10$$

$$R4 := 10 \quad R2 := 0.1 \quad TP := 3\cdot 10^{-3} \quad DT := \frac{TP}{4000} \quad DT = 7.5\times 10^{-7}$$

where TP – the transient account time which is equal to five of transient constant time

Determining of independent initial conditions under the commutation laws

$$IO := \frac{E}{(R1+R3+R4)} \quad IO = 0.995 \quad UCO := IO\cdot(R3+R4) \quad UCO = 19.9$$

The assignment values of initial condition to the state vector

$$x := \begin{pmatrix} UCO \\ IO \end{pmatrix}$$

Matrix of derivative states variable on energy storages

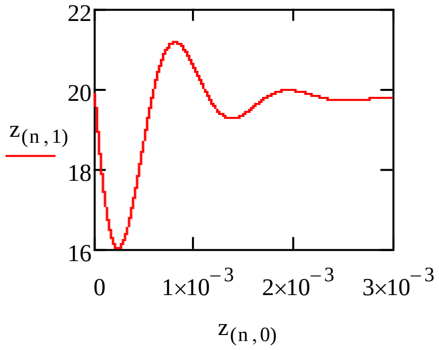
$$D(t,x) := \begin{bmatrix} \dfrac{x_1\cdot R3 - x_0}{C\cdot(R2+R3)} \\ E\cdot\dfrac{1}{L} - \dfrac{1}{L}\cdot x_0 - \dfrac{R1}{L}\cdot x_1 - \dfrac{\left(x_1\cdot R3 - x_0\right)\cdot R2}{L\cdot(R2+R3)} \end{bmatrix}$$

Digital integrating the derivative states variable by the Runge-Kutt method with constant integrating step t DT

$$z := \mathrm{rkfixed}\left(x, 0, TP, \frac{TP}{DT}, D\right)$$

$$n := 0..4000$$

Capacitance voltage and induction current in the functions of time



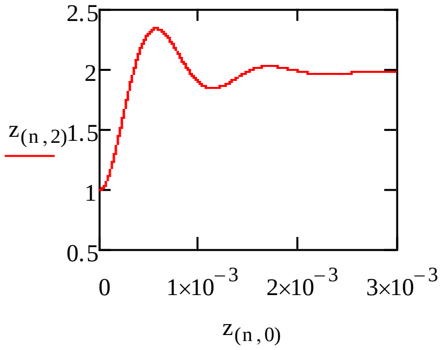


Fig. 3.8.

3.5. The circuit parameters calculation of transients at action in the circuit of power supply with the arbitrary form of output signal

At the power supply output signal arbitrary form calculation is performed on basis of the circuit input signal piecewise-linear approximation and the reaction search of circuit with using of Duhamel integral.

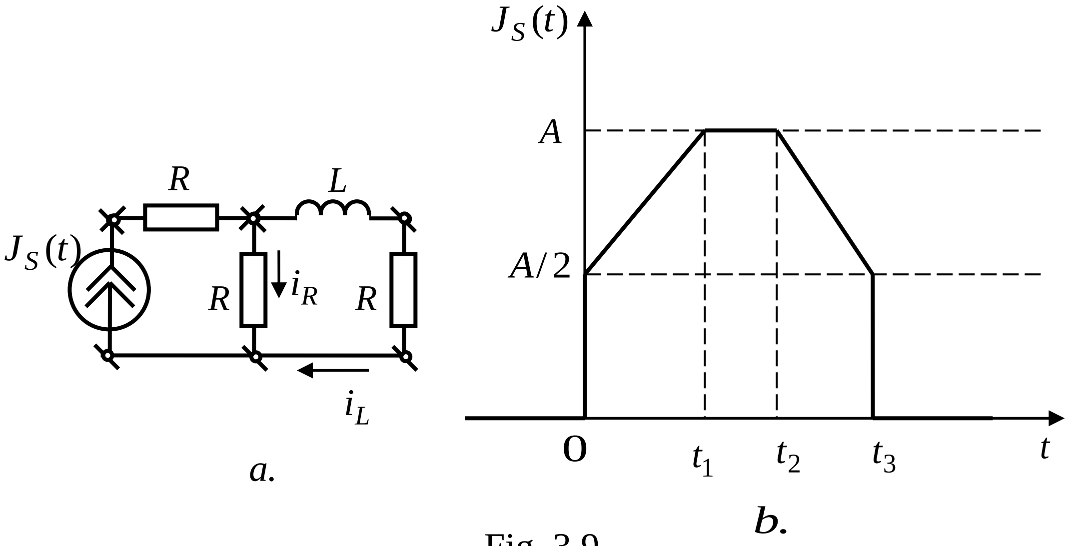


Fig. 3.9

Task.
It is Given electric circuit, Fig. 3.9.a, on input which acts current source, output signal of one $J_S(t)$ changes according to given law Fig. 3.9.b. There is necessary determining law of current and voltage variation in the circuit branches

Task solving.
In order to take advantage of Duhamel integral there is necessary described the circuit input current. The output signal of current source, according to the graph Fig. 3.9.b

$$\left.\begin{aligned}
&J_{S1}(t)=0, npu_t<0;\\
&J_{S2}(t)=\frac{A}{2}\left(1+\frac{t}{t_1}\right), npu_0\le t<t_1;\\
&J_{S3}(t)=A, npu_t_1\le t<t_2;\\
&J_{S4}(t)=A+\left(\frac{A}{2}-A\right)\frac{t-t_2}{t_3-t_2}, npu_t_2\le t<t_3;\\
&J_{S5}(t)=0, npu_t\ge t_3.
\end{aligned}\right\}$$

We find transients by classical approach

$$i_R(t)=J_S(t)-i_L(t)=J_S(t)-\frac{J_S(t)}{2}\left(1-e^{-\frac{L}{2R}t}\right)=\frac{J_S(t)}{2}\left(1+e^{-\frac{L}{2R}t}\right).$$

Normolized transient functions find from the action of current source with output current single value ($J_S(t)$=1, A)

$$i_L(t)=\frac{J_S(t)}{2}\left(1-e^{-\frac{L}{2R}t}\right);u_L(t)=L\frac{di_L(t)}{dt}=\frac{L}{4R}e^{-\frac{L}{2R}t};$$

$$h_1(t)=\frac{i_L(t)}{J_S(t)}=\frac{1}{2}\left(1-e^{-\frac{L}{2R}t}\right);h_2(t)=\frac{u_L(t)}{J_S(t)}=\frac{L}{4R}e^{-\frac{L}{2R}t};h_3(t)=\frac{i_R(t)}{J_S(t)}=\frac{1}{2}\left(1+e^{-\frac{L}{2R}t}\right).$$

We shall notice that the normolized transient functions $h_1(t),h_3(t)$ dimensionless, function $h_2(t)$ has of dimension of resistance, that is why it is called by transient resistance which changes in the time functions in this case according to exponential law.

We find of electric circuit currents and voltages on the sections of piecewise-linear approximation input current with using of Duhamel integral

$$y(t)=h(t)\cdot J_S(0)+\int_0^t h(t-x)J_S'(x)dx,$$

where $y(t)$– desired quantity circuit branch reaction; x – integrating parameter; $h(t-x)$ – shifted normolized transient function; $J_S'(x)$ – derivative from input signal over the integrating parameter *x*.

For researched circuit Fig. 4.2, having in the time period $0\le t<t_1$ source current in *t=0* stepwise increases from null value till value *A/2*, and then linearly increases till value *A* as to given law $J_{S2}(t)$

$$0\le t<t_1;J_S(t)=\frac{A}{2}(1+\frac{t}{t_1});$$

$$i_L(t)=h_1(t)\cdot J_{S2}(0)+\int_0^t h_1(t-x)J_{S2}'(x)dx=\frac{A}{2}\cdot\frac{1}{2}\left(1-e^{-\frac{L}{2R}t}\right)+$$

$$+\int_0^t\frac{1}{2}\left(1-e^{-\frac{L}{2R}(t-x)}\right)\frac{A}{2t_1}dx=\frac{A}{4}\cdot\left(1-e^{-\frac{L}{2R}t}\right)+\frac{A}{4t_1}t+\frac{1}{2}e^{-\frac{L}{2R}(t)}\frac{2R}{L}\frac{A}{2t_1}(e^{\frac{L}{2R}(t)}-1)=$$

$$=\frac{A}{4}\cdot\left(1-e^{-\frac{L}{2R}t}\right)+\frac{A}{4t_1}t+\frac{1}{2}\frac{2R}{L}\frac{A}{2t_1}(1-e^{-\frac{L}{2R}(t)});$$

On the second approximation stage $t_1\le t<t_2$ input current is given by function $J_{S3}(t)$ and continues the circuit reaction on influence $J_{S2}(t)$

$$i_L(t)=h_1(t_1)\cdot J_{S2}(0)+\int_0^{t_1} h_1(t-x)J'_{S2}(x)dx+h_1(t-t_1)\cdot(J_{S3}(t_1)-J_{S2}(t_1))+$$

$$+\int_{t_1}^{t_2} h_1(t-x)J'_{S3}(x)dx=\frac{A}{4}\cdot\left(1-e^{-\frac{L}{2R}t_1}\right)+\frac{A}{4}+\frac{1}{2}\frac{2R}{L}\frac{A}{2t_1}(1-e^{-\frac{L}{2R}(t_1)}).$$

Because is absent the stepwise action at the time moment $t=t_1$ and does not change input action on time $t_1\le t<t_2$ that circuit reaction is determined by action which was on the previous interval of approximation.

On the third section of approximation input signal begins to decrease from value *A* till value *A/2* under the law $J_{S3}(t)$

$$i_L(t)=h_1(t_1)\cdot J_{S2}(0)+\int_0^{t_1} h_1(t-x)J'_{S2}(x)dx+$$

$$+h_1(t-t_1)\cdot(J_{S3}(t_1)-J_{S2}(t_1))+\int_{t_1}^{t_2} h_1(t-x)J'_{S3}(x)dx+$$

$$+h_1(t-t_2)\cdot(J_{S4}(t_2)-J_{S3}(t_2))+\int_{t_2}^{t} h_1(t-x)J'_{S4}(x)dx=$$

$$=\frac{A}{4}\cdot\left(1-e^{-\frac{L}{2R}t_1}\right)+\frac{A}{4}+\frac{1}{2}\frac{2R}{L}\frac{A}{2t_1}(1-e^{-\frac{L}{2R}(t_1)})+\left(-\frac{A}{4}\frac{1}{t_3-t_2}\right)(t-t_2)+\frac{1}{2}\frac{2R}{L}e^{-\frac{L}{2R}t-t_2}.$$

On the fourth section of approximation continue to act current and voltage transient functions from action at previous time sections and add negative current stepwise at the moment $t=t_3$

$$i_L(t)=h_1(t_1)\cdot J_{S2}(0)+\int_0^{t_1} h_1(t-x)J'_{S2}(x)dx+$$

$$+h_1(t-t_1)\cdot(J_{S3}(t_1)-J_{S2}(t_1))+\int_{t_1}^{t_2} h_1(t-x)J'_{S3}(x)dx+$$

$$+h_1(t-t_2)\cdot(J_{S4}(t_2)-J_{S3}(t_2))+\int_{t_2}^{t_3} h_1(t-x)J'_{S4}(x)dx+$$

$$+h_1(t-t_3)\cdot(J_{S5}(t_2)-J_{S4}(t_2))+\int_{t_3}^{t} h_1(t-x)J'_{S5}(x)dx=$$

$$=\frac{A}{4}\cdot\left(1-e^{-\frac{L}{2R}t_1}\right)+\frac{A}{4}+\frac{1}{2}\frac{2R}{L}\frac{A}{2t_1}(1-e^{-\frac{L}{2R}(t_1)})+\left(-\frac{A}{4}\frac{1}{t_3-t_2}\right)(t_3-t_2)+\frac{1}{2}\frac{2R}{L}e^{-\frac{L}{2R}t_3-t_2}-$$

$$-\frac{A^2}{8}\cdot\left(1-e^{-\frac{L}{2R}(t-t_3}\right).$$

The branches reactions rest are calculated similarly as to mentioned algorithm.

3.6. The personal computative-graphic task "The calculation of transients in linear circuits"

The personal job consists of three tasks. The first task envisages the calculation of transient function by classical approach in circuits with have two energy storages, second task – calculation this transient function by operational method and third – calculation of transient function in circuits with one energy storage at arbitrary form of energy source output signal.

Tasks 1, 2. B In electric circuit Fig.3.10-3.29 perform commutation. In circuit there is act DC EMF *E*. The parameters of circuit are given in Table 3.1. Is needed determine in time the law of variation transient function after commutation moment in one of the scheme branche. Task to decide by two methods: classic and operational. On the grounds of finded analytic expression for transient function to build the graph of the found value change on interval beginning from the commutation moment and till the value of time determined 5 maximal time constant, when natural component to decline till 99% from initial value.

Guidance.

1. Equation for the operational images of scheme Fig.3.11 is recommended compose as to the method of node potentials with allowance for having in the scheme of energy sources and unnull initial condition.

2. In scheme Fig.3.20, with the purpose of the composing simplification of characteristically equation and equations for operational images the left scheme contour E, R_1, R_2, R_3 is recommended in calculations to replace by equivalent power supply with inner ideal power supply and inner resistance.

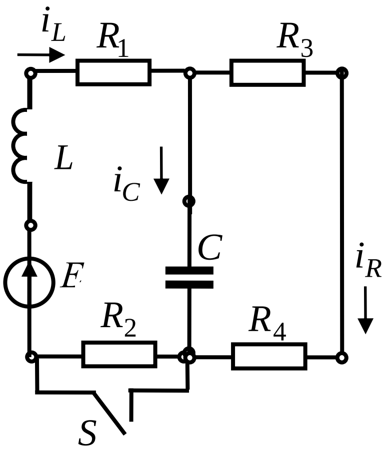


Fig.3.10

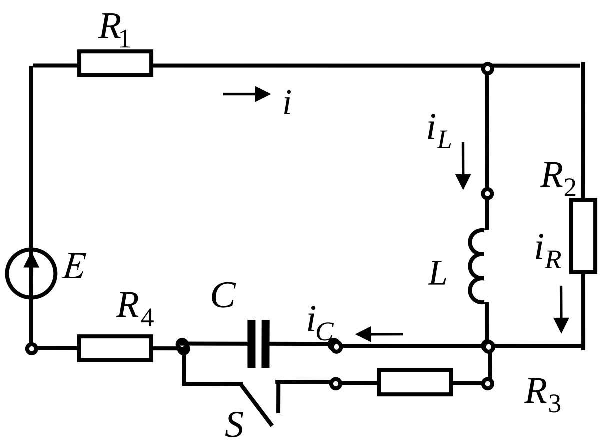


Fig.3.11

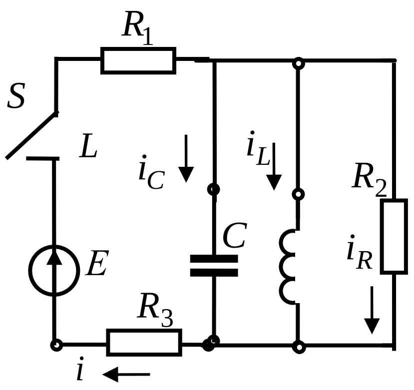


Fig.3.12

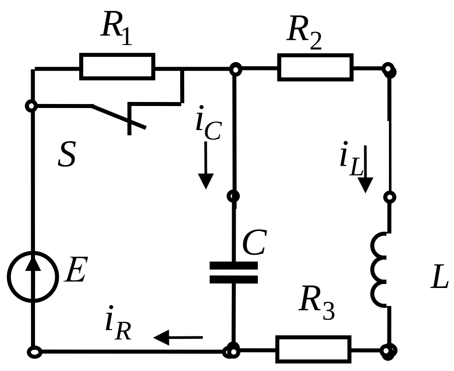


Fig.3.13

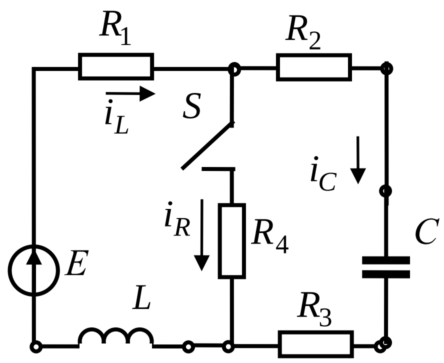


Fig.3.14

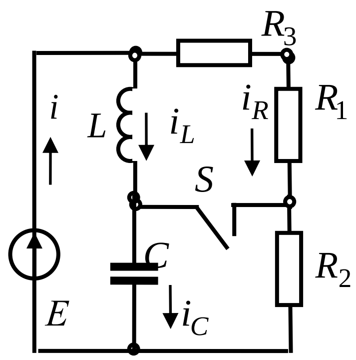


Fig. 3.15

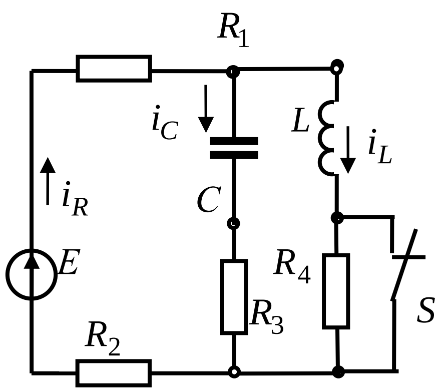


Fig. 3.16

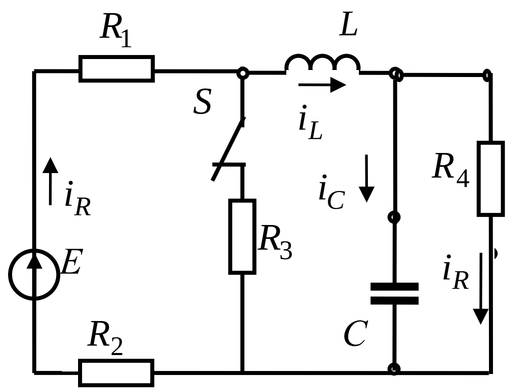


Fig. 3.17

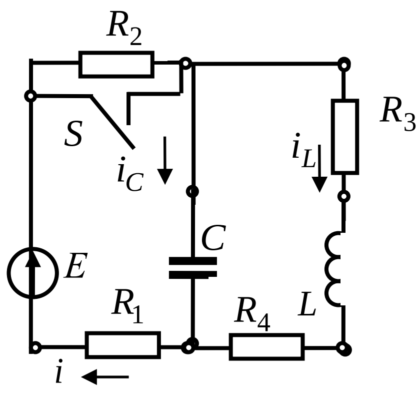


Fig.3.18

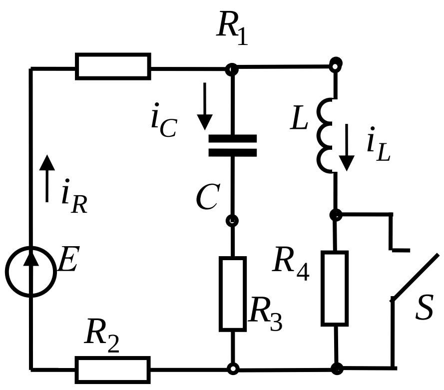


Fig.3.19

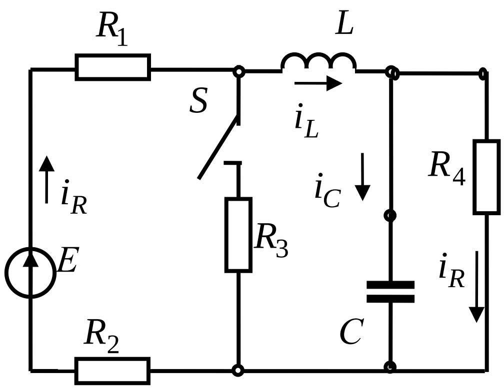


Fig. 3.20

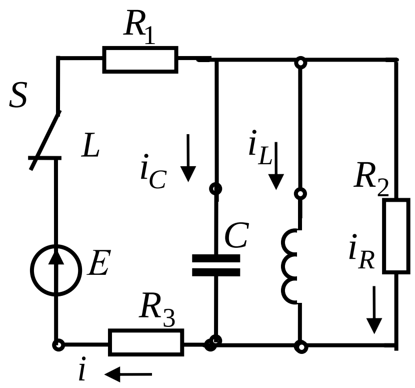


Fig.3.21

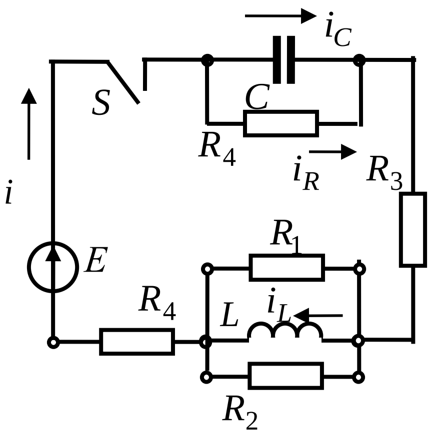


Fig. 3.22

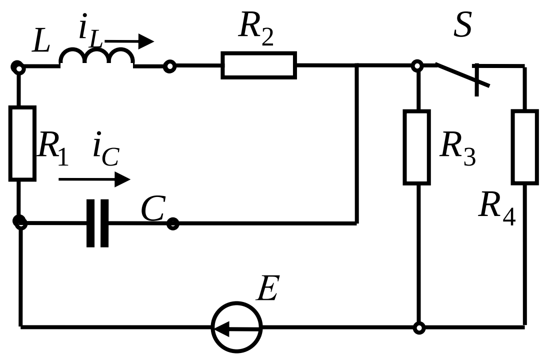


Fig. 3.23

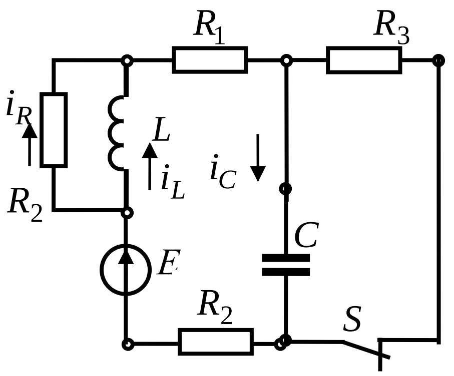


Fig. 3.24

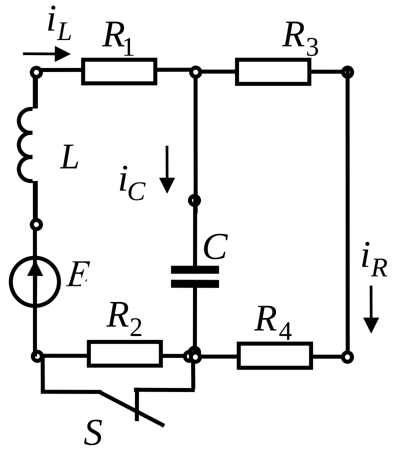


Fig.3.25

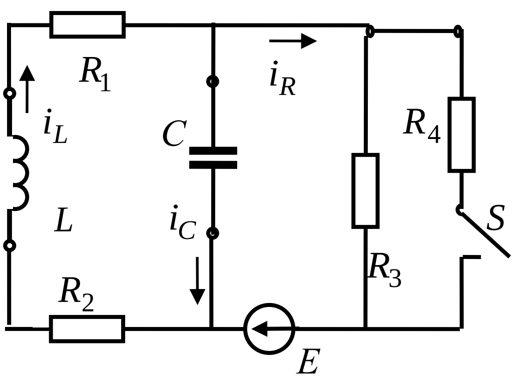


Fig. 3.26

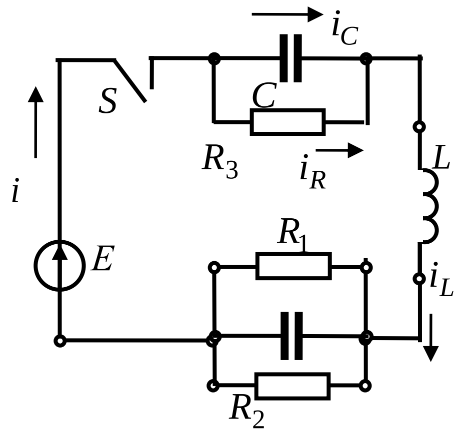


Fig. 3.27

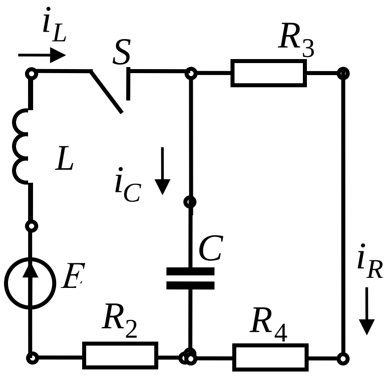


Fig. 3.28

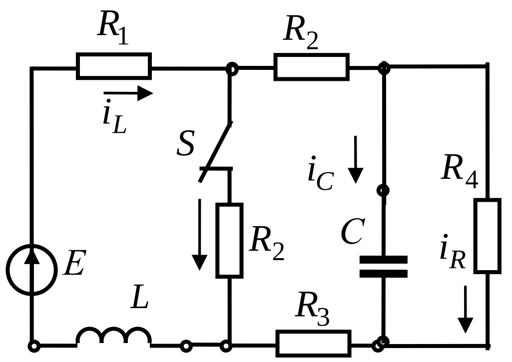


Fig. 3.29

Task 3. There is given electric scheme, Fig.3.30-3.35, on input one which acts voltage that changed in time as to known law $u_1(t)$. Is needed to determine law of current variation in one of the scheme branches or variation voltage on the given scheme district . In Table 3.2 according to the number of variant is specified the number of picture, on which is showing the input voltage graph change in the time, Fig.3.36-3.45. The circuit parameters R, L, C are given generally type.

Task is needed to decide with the help of Duhamel integral . Desired quantity follows to determine (to write down its analytic expression) for all time tintervals. Depending on statement of problems complete answer will comprise two or three summand, each of which is correct only in the definite borders of the change of time t.

In every answer follows to fulfil the adduction of similar members relatively of coefficients $e^{b_1 t}, e^{b_2 t}, t$ and separate the constant component.

Note. On Fig.3.40, 3.41, 3.45 input voltage is given with two indexes. The first index (index 1) points at input voltage, second index (1 or 2) _ on the time interval to which belongs input voltage. It is so, for example, u_{11} – input voltage for the first time interval, u_{12} – input voltage for the second time interval.

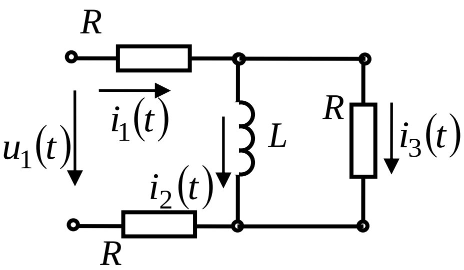


Fig. 3.30

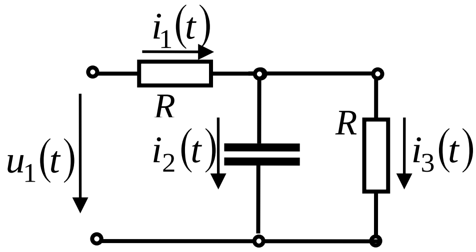


Fig. 3.31

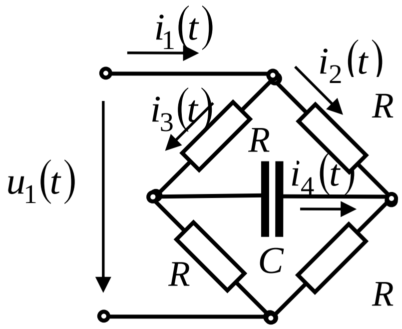


Fig. 3.32

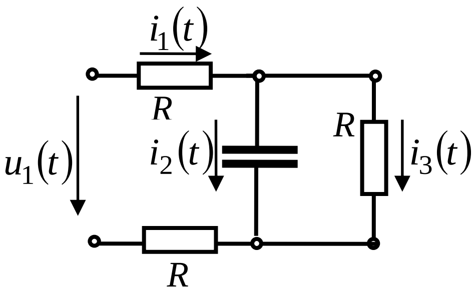
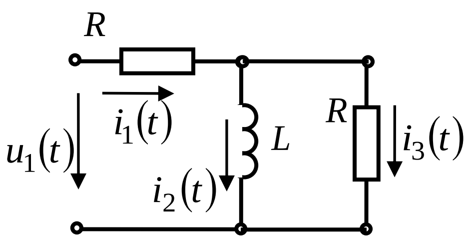


Fig. 3.33



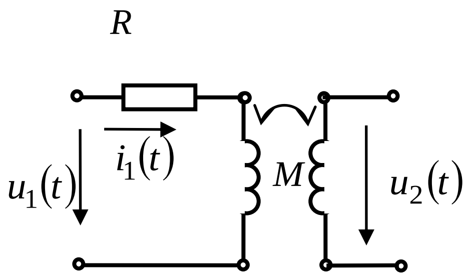


Fig. 3.34

Fig. 3.35

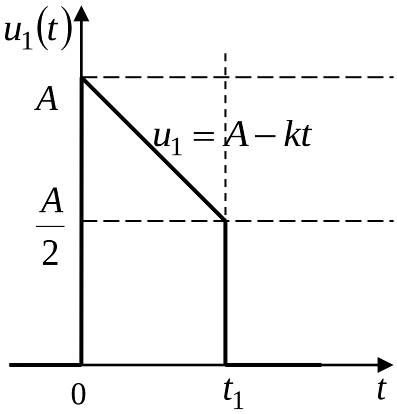


Fig. 3.36

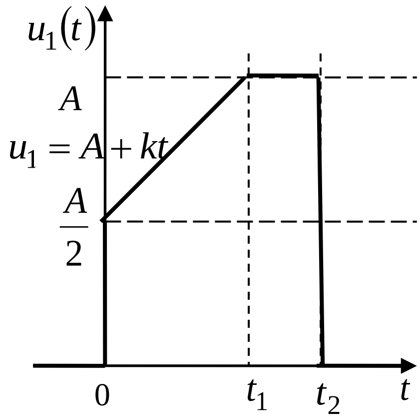


Fig. 3.37

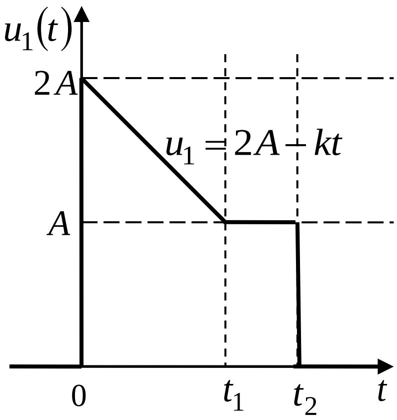


Fig. 3.38

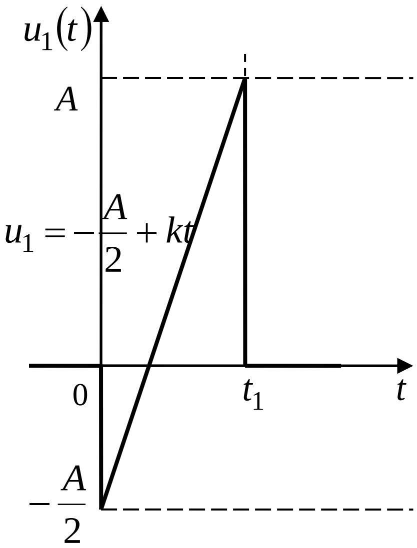


Fig. 3.39

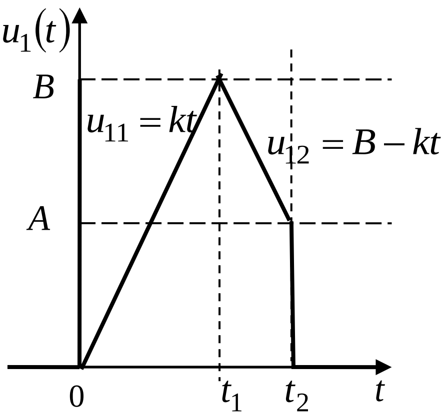


Fig. 3.40

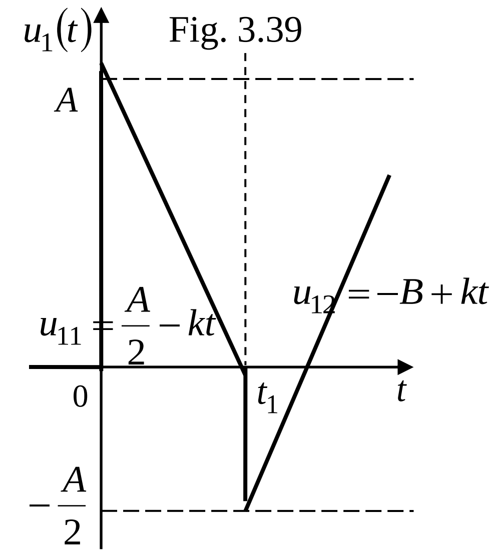


Fig. 3.31

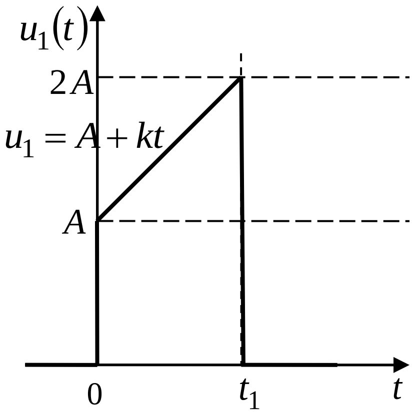


Fig. 3.42

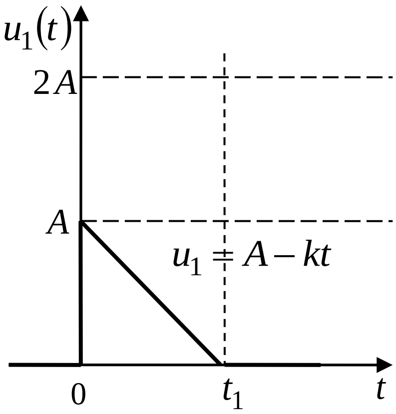


Fig. 3.43

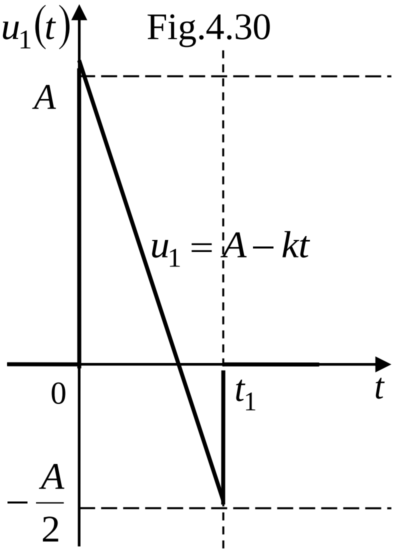


Fig. 3.44

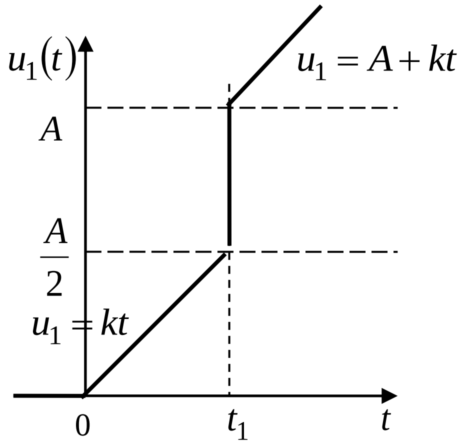


Fig. 3.45

Table 3.1

Variant	Figure	E, V	L, mH	C, μF	R_1, ohm	R_2, ohm	R_3. ohm	R_4, ohm	Define
01	3.14	100	1	10	20	15	5	2	i
02	3.11	150	2	5	8	10	5	2	i_L
03	3.28	100	1	10	2	2	0	0	i
04	3.19	120	1	10	3	0	1	1	i_C
05	3.12	100	5	50	2	8	6	0	i
06	3.10	50	1	1500	2	13	1	4	i_R
07	3.20	120	10	10	10	90	1000	1000	i_L
08	3.27	200	1	20	4	4	2	0	i_R
09	3.13	100	1	10	50	25	25	0	u_C
10	3.26	300	5	4	10	20	10	20	u_C

Continue of Table 3.1

Variant	Figure	E, V	L, mH	C, μF	R_1, ohm	R_2, ohm	R_3. ohm	R_4, ohm	Define
11	3.29	100	1	10	20	4	16	2	u_{R2}
12	3.24	150	4	5	6	10	5	4	u_C
13	3.15	30	1	265	10	10	10	0	u_C
14	3.16	200	10	10	100	0	50	100	i_L
15	3.21	100	1	10	10	10	4	0	i
16	3.25	50	2	1670	1	2	1	5	i
17	3.17	120	10	10	10	90	1000	1000	i_L
18	3.22	120	1	10	8	8	8	4	i_C
19	3.18	200	1	10	10	20	50	20	i_L
20	3.23	50	1	100	2	8	10	10	i_L
21	3.14	100	1	10	20	20	0	2	u_L
22	3.11	150	2	5	5	10	5	5	i_C
23	3.28	100	1	10	1	3	0	0	i_R
24	3.19	120	1	10	1	2	1	1	i_L
25	3.12	100	5	50	3	8	5	0	u_C
26	3.10	50	1	1500	2	13	2	3	i_L
27	3.20	120	10	10	20	80	1000	1000	i_C
28	3.27	200	1	20	6	3	2	0	i_L
29	3.13	100	1	10	50	20	30	0	u_L
30	3.26	300	5	4	15	20	5	20	i_C
31	3.29	100	1	10	20	17	3	2	i_L
32	3.24	150	4	5	9	10	5	1	u_L
33	3.15	30	1	2,5	5	10	15	0	i_L
34	3.16	200	10	10	50	50	50	100	u_{R3}
35	3.21	100	1	10	5	15	4	0	u_L
36	3.25	50	2	1670	1	2	3	3	u_{R2}
37	3.17	120	10	10	20	80	1000	1000	i_C
38	3.22	120	1	10	12	6	8	4	i_L
39	3.18	200	1	10	10	10	50	30	i_C
41	3.14	100	1	10	20	2	18	2	u_C
42	3.11	150	2	5	4	10	5	6	i_R
43	3.28	100	1	10	1,5	2,5	0	0	i_C

Continue of Table 3.1

Variant	Figure	E, V	L, mH	C, μF	R_1, ohm	R_2, ohm	R_3. ohm	R_4, ohm	Define
44	3.19	120	1	10	2	1	1	1	u_{R2}
45	3.12	100	5	50	6	8	2	0	i_L
46	3.10	50	1	1500	2	13	3	2	u_L
47	3.20	120	10	10	30	70	1000	1000	i_R
48	3.27	200	1	20	12	2,4	2	0	i_C
49	3.13	100	1	10	50	10	40	0	i_L
50	3.26	300	5	4	3	20	17	20	i_L
51	3.29	100	1	10	20	8	12	2	u_L
52	3.24	150	4	5	0	10	5	10	i
53	3.15	30	1	2,5	15	10	5	0	i
54	3.16	200	10	10	25	75	50	100	u_C
55	3.21	100	1	10	15	5	4	0	i_C
56	3.25	50	2	1670	1	2	3	3	u_L
57	3.17	120	10	10	30	70	1000	1000	i_R
58	3.22	120	1	10	24	4,8	8	4	i_R
59	3.18	200	1	10	10	25	50	15	i
60	3.23	50	1	100	4	6	10	10	i_R
61	3.14	100	1	10	20	10	10	2	u_C
62	3.11	150	2	5	7	10	5	3	u_L
63	3.28	100	1	10	3	1	0	0	u_L
64	3.19	120	1	10	1,5	1,5	1	1	u_L
65	3.12	100	5	50	3	8	5	0	u_C
66	3.10	50	1	1500	2	13	4	1	i
67	3.20	120	10	10	40	60	1000	1000	u_L
68	3.27	200	1	20	3	6	2	0	u_L
69	3.13	100	1	10	50	30	20	0	i
70	3.26	300	5	4	6	20	14	20	u_L
71	3.29	100	1	10	20	11	9	2	u_C
72	3.24	150	4	5	3	10	5	7	i_C
73	3.15	30	1	2,5	12	10	8	0	i_C
74	3.16	200	10	10	0	100	50	100	u_L
75	3.21	100	1	10	15	5	4	0	i_C
76	3.25	50	2	1670	1	2	4	2	u_C

Continue of Table 3.1

Variant	Figure	E, V	L, mH	C, μF	R_1, ohm	R_2, ohm	R_3. ohm	R_4, ohm	Define
77	3.17	120	10	10	40	60	1000	1000	u_L
78	3.22	120	1	10	6	12	8	4	u_C
79	3.18	200	1	10	10	30	50	10	u_L
80	3.23	50	1	100	5	5	10	10	u_L
81	3.14	100	1	10	20	16	4	2	u_{R4}
82	3.11	150	2	5	10	10	5	0	u_C
83	3.28	100	1	10	4	0	0	0	u_C
84	3.19	120	1	10	0	3	1	1	u_C
85	3.12	100	5	50	4	8	4	0	u_L
86	3.10	50	1	1500	2	13	5	0	u_{R1}
87	3.20	120	10	10	50	50	1000	1000	u_C
88	3.27	200	1	20	4	4	2	0	u_C
89	3.13	100	1	10	50	35	15	0	i_C
90	3.26	300	5	4	4	20	16	20	u_{R1}
91	3.29	100	1	10	20	13	7	2	i_C
92	3.24	150	4	5	2	10	5	8	u_{R1}
93	3.15	30	1	2,5	8	10	12	0	u_L
94	3.16	200	10	10	75	25	50	100	u_{R3}
95	3.21	100	1	10	13	7	4	0	u_{R3}
96	3.25	50	2	1670	1	2	5	1	u_{R1}
97	3.17	120	10	10	50	50	1000	1000	u_C
98	3.22	120	1	10	8	8	8	4	u_L
99	3.18	200	1	10	10	18	50	22	u_C
00	3.23	50	1	100	6	4	10	10	u_C

Table 3.2

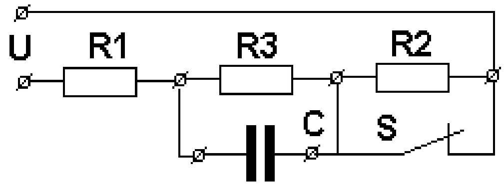
Variant	Scheme	Input voltage graph	Define	Variant	Scheme	Input voltage graph	Define
01	3.32	3.36	$i_4(t)$	36	3.33	3.38	$i_2(t)$
02	3.31	3.36	$Ri_1(t)$	37	3.32	3.39	$u_C(t)$
03	3.33	3.36	$Ri_3(t)$	38	3.35	3.45	$u_2(t)$
04	3.32	3.38	$i_1(t)$	39	3.32	3.43	$i_2(t)$
05	3.30	3.36	$i_1(t)$	40	3.32	3.37	$i_3(t)$
06	3.31	3.37	$i_2(t)$	41	3.32	3.40	$i_4(t)$
07	3.33	3.40	$i_1(t)$	42	3.31	3.44	$Ri_1(t)$
08	3.34	3.36	$i_1(t)$	43	3.31	3.40	$Ri_3(t)$
09	3.34	3.39	$i_2(t)$	44	3.34	3.44	$i_1(t)$
10	3.30	3.37	$u_L(t)$	45	3.30	3.40	$i_1(t)$
11	3.33	3.36	$Ri_3(t)$	46	3.31	3.44	$i_2(t)$
12	3.31	3.36	$i_3(t)$	47	3.33	3.44	$i_1(t)$
13	3.34	3.36	$i_3(t)$	48	3.34	3.44	$i_1(t)$
14	3.31	3.36	$u_C(t)$	49	3.34	3.45	$i_2(t)$
15	3.34	3.37	$u_L(t)$	50	3.30	3.39	$u_L(t)$
16	3.33	3.37	$i_2(t)$	51	3.33	3.42	$Ri_3(t)$
17	3.32	3.43	$u_C(t)$	52	3.31	3.45	$i_3(t)$
18	3.35	3.40	$u_2(t)$	53	3.34	3.42	$i_3(t)$
19	3.32	3.36	$i_2(t)$	54	3.31	3.44	$u_C(t)$
20	3.32	3.36	$i_3(t)$	55	3.34	3.39	$u_L(t)$
21	3.32	3.37	$i_4(t)$	56	3.31	3.39	$i_2(t)$
22	3.31	3.39	$Ri_1(t)$	57	3.44	3.40	$u_C(t)$
23	3.33	3.39	$Ri_3(t)$	58	3.35	3.42	$u_2(t)$
24	3.32	3.39	$i_1(t)$	59	3.32	3.40	$i_2(t)$
25	3.30	3.37	$i_1(t)$	60	3.32	3.39	$i_3(t)$
26	3.31	3.38	$i_2(t)$	61	3.32	3.38	$i_4(t)$
27	3.33	3.41	$i_1(t)$	62	3.31	3.45	$Ri_1(t)$
28	3.34	3.40	$i_1(t)$	63	3.33	3.41	$Ri_3(t)$
29	4.34	3.41	$i_2(t)$	64	3.32	3.45	$i_1(t)$
30	3.30	3.38	$u_L(t)$	65	3.30	3.39	$i_1(t)$
31	3.33	3.39	$Ri_3(t)$	66	3.31	3.45	$i_2(t)$
32	3.31	3.44	$i_3(t)$	67	3.33	3.45	$i_1(t)$

Continue of Table 3.2

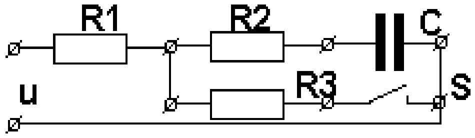
Variant	Scheme	Input voltage graph	Define	Variant	Scheme	Input voltage graph	Define
33	3.34	3.39	$i_3(t)$	68	3.34	3.45	$i_1(t)$
34	3.31	3.39	$u_C(t)$	69	3.34	3.42	$i_2(t)$
35	3.34	3.38	$u_L(t)$	70	3.30	3.42	$u_L(t)$
71	3.33	3.43	$Ri_3(t)$	86	3.31	3.42	$i_2(t)$
72	3.31	3.42	$i_3(t)$	87	3.33	3.42	$i_1(t)$
73	3.34	3.44	$i_3(t)$	88	3.34	3.42	$i_1(t)$
74	3.31	3.40	$u_C(t)$	89	3.34	3.43	$i_2(t)$
75	3.34	3.40	$u_L(t)$	90	3.30	3.44	$u_L(t)$
76	3.33	3.40	$i_2(t)$	91	3.33	3.40	$Ri_3(t)$
77	3.32	3.44	$u_C(t)$	92	3.31	3.43	$i_3(t)$
78	3.35	3.43	$u_2(t)$	93	3.34	3.45	$i_3(t)$
79	3.32	3.41	$i_2(t)$	94	3.31	3.42	$u_C(t)$
80	3.32	3.42	$i_3(t)$	95	3.34	3.41	$u_L(t)$
81	3.32	3.43	$i_4(t)$	96	3.33	3.41	$i_2(t)$
82	3.31	3.43	$Ri_1(t)$	97	3.32	3.36	$u_C(t)$
83	3.33	3.44	$Ri_3(t)$	98	3.35	3.36	$u_2(t)$
84	3.32	3.42	$i_1(t)$	99	3.32	3.45	$i_2(t)$
85	3.30	3.45	$i_1(t)$	00	3.32	3.43	$i_3(t)$

3.7. Questions for one's own checking as to the calculation methods of transients in linear circuits

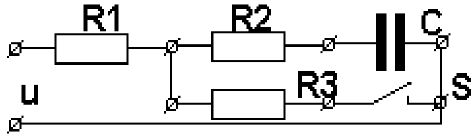
1. Find current value across capacitor at moment of commutation (t=0), if applied voltage is U=80 V, and R1=2 Ω, R2=8Ω, R3=6 Ω.



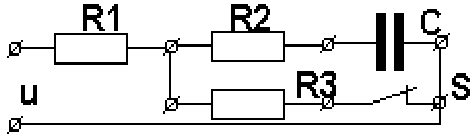
2. Find current value through resister R2 at moment of commutation (t=0), if U=150 B, R1=10 Ω, R2=R3=5 Ω.



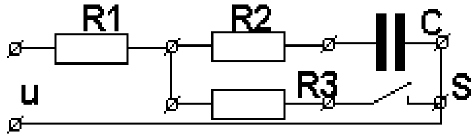
3. Find current value through resistor R2 at moment of commutation (t=0), if U=150 V, R1=10 Ω, R2=5 Ω, R3=5 Ω.



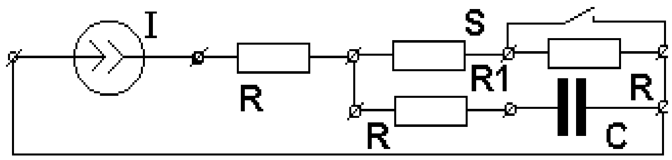
4. Find current value through resistor R1 at moment of commutation (t=0), if U=150 V, R1=10 Ω,R2=5 Ω, R3=5Ω.



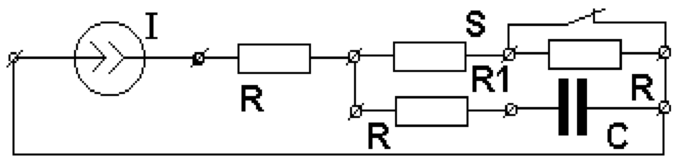
5. Find current value through resister R1 at moment of commutation (t=0), if U=150 V, R1=10 Ω, R2=5 Ω, R3=5 Ω.



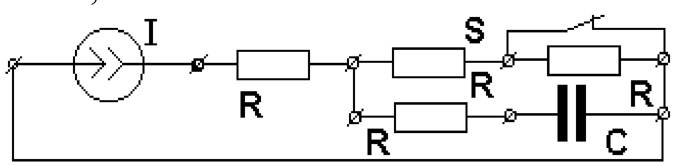
6. Find current value through resister R1 at moment of commutation (t=0), if I=10 A, R=R1=10 Ω.



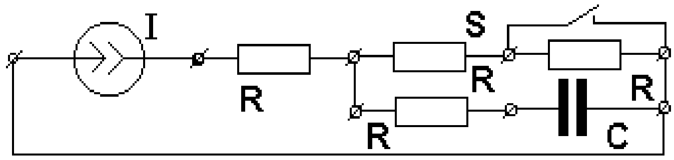
7. Find current value through resister R1 at moment of commutation (t=0), if I=10 A, R1=R=10 Ω.



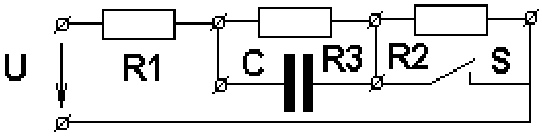
8. Find current value through capacitor at moment of commutation (t=0), if applied current is I=10 A; and R=10 Ω.



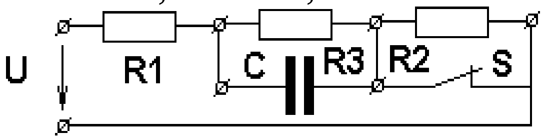
9. Find current value through capacitor at moment of commutation (t=0), if applied current is I=10 A; and R=10 Ω.



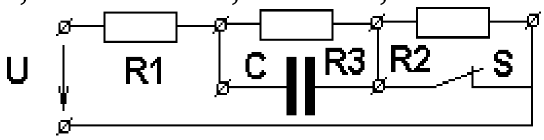
10. Find current value through capacitor at moment of commutation (t=0), if applied voltage is U=80 V, and R1=2 Ω, R2=8 Ω, R3=6 Ω.



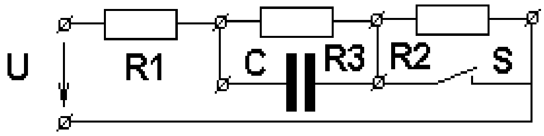
11. Find current value through capacitor at moment of commutation (t=0), if applied voltage is U=80 V, and R1=2 Ω, R2=8 Ω, R3=6 Ω.



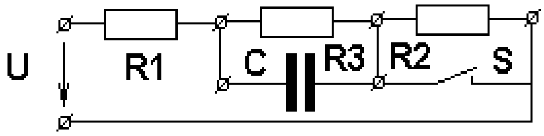
12. Find current value through resistor R3 at moment of commutation (t=0), if applied voltage is U=80 V, and R1=2 Ω, R2=8 Ω, R3=6 Ω.



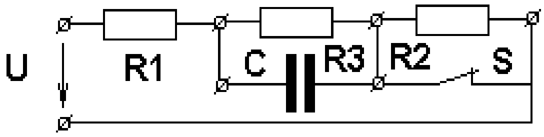
13. Find current value through resistor R3 at moment of commutation (t=0), if applied voltage is U=80 V, and R1=2 Ω, R2=8 Ω, R3=6 Ω.



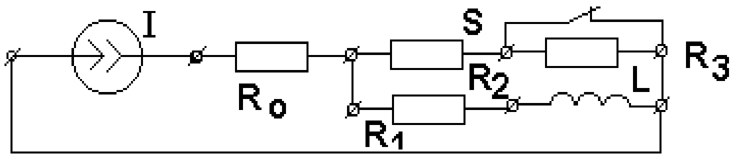
14. Find current value through resistor R1 at moment of commutation (t=0), if applied voltage is U=80 V, and R1=2 Ω, R2=8 Ω, R3=6 Ω.



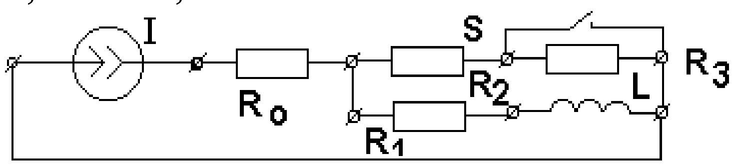
15. Find current value across resistor R1 at moment of commutation (t=0), if applied voltage is U=80 V, and R1=2 Ω, R2=8 Ω, R3=6 Ω.



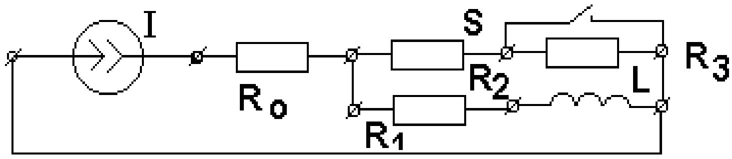
16. Define current value across resistor R2 at moment of commutation (t=0), if I=1 A, R0=R2=2 Ω, R1=8 Ω, R3=90 Ω.



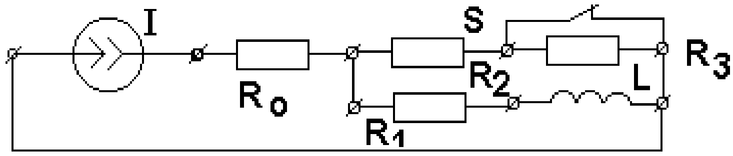
17. Define current value through resistor R2 at moment of commutation (t=0), if I=1 A, R0=R2=2 Ω, R1=8 Ω, R3=90 Ω.



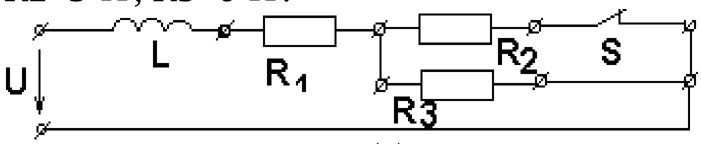
18. Find voltage value across inductance $u_L(0)$ at moment of commutation (t=0), if I=1 A, R0=R2=2 Ω, R1=8 Ω, R=3,9 Ω.



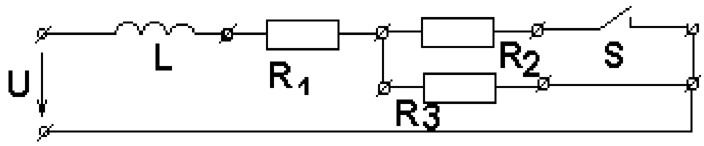
19. Find voltage value across inductance $u_L(0)$ at moment of commutation (t=0), if I=1 A, R0=R2=2Ω, R1=8 Ω, R3=8 Ω.



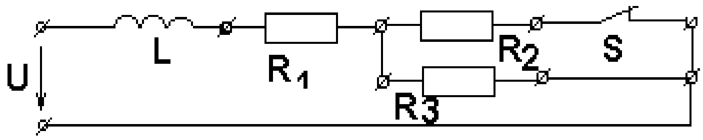
20. Find voltage value over inductance $u_L(0)$ at moment of commutation (t=0), if U=160 V, R1=8Ω, R2=3 Ω, R3=6 Ω.



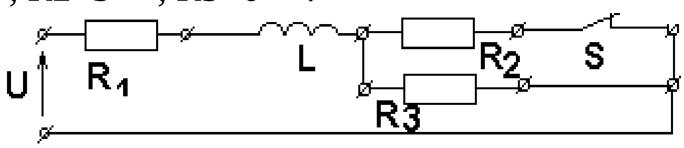
21. Find voltage value across inductance $u_L(0)$ at moment of commutation (t=0), if U=160 V, R1=8Ω, R2=3 Ω, R3=6 Ω.



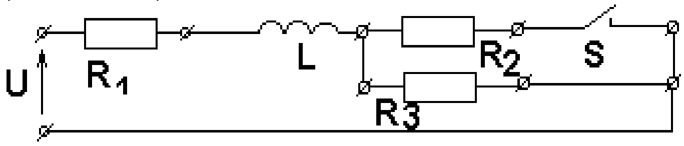
22. Define current value through resistor R1 at moment of commutation (t=0), if U=160 V, R1=8 Ω, R2=3 Ω, R3=6 Ω.



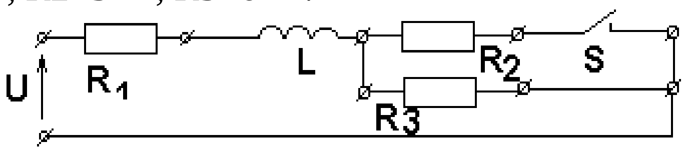
23. Define current value through resistor R3 at moment of commutation (t=0), if U=160 V, R1=8 Ω, R2=3 Ω, R3=6 Ω.



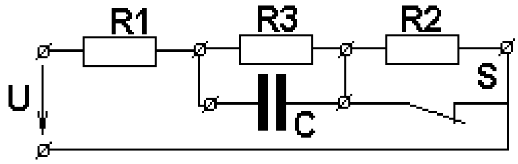
24. Define current value through resistor R3 at moment of commutation (t=0), if U=160 V, R1=8 Ω, R2=3 Ω, R3=6 Ω.



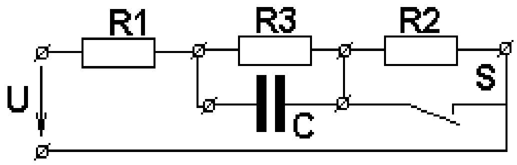
25. Define current value through resistor R2 at moment of commutation (t=0), if U=160 V, R1=8 Ω, R2=3 Ω, R3=6 Ω.



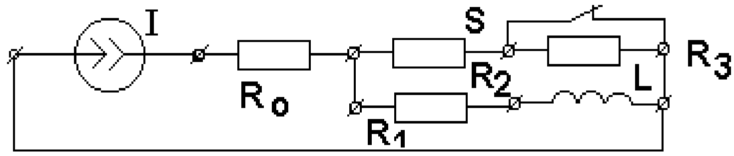
26. Find the constant time τ of transient process if given: R1=2 Ω, R2=8 Ω, R3=6 Ω, C=10 μF .



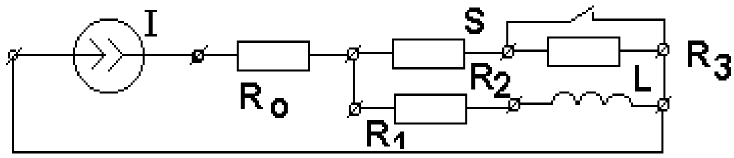
27. Find the constant time τ of transient process if given: R1=2 Ω, R2=8 Ω, R3=6 Ω, C=10 μF .



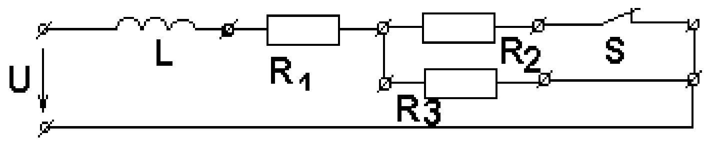
28. Find the constant time τ of transient process if given: R0=R2=2 Ω, R1=8 Ω, R3=90 Ω, L=100 mH.



29. Find the constant time τ of transient process if given: R0=R2=2 Ω, R1=8 Ω, R3=90 Ω, L=100 mH.

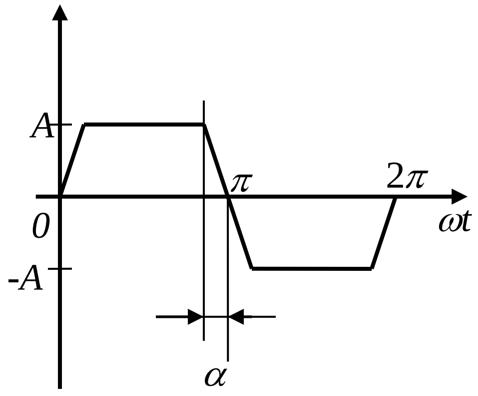
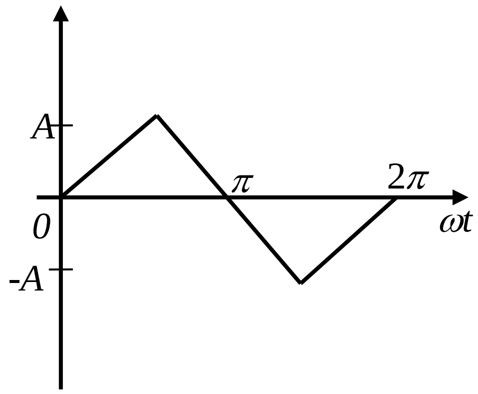
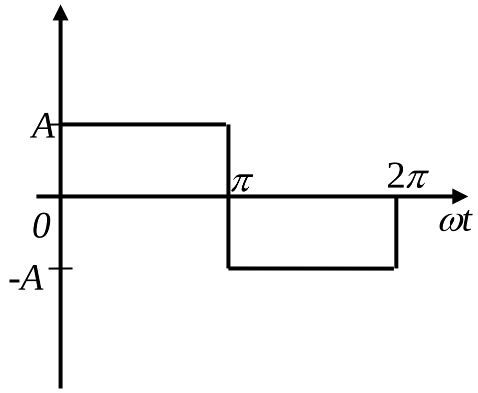


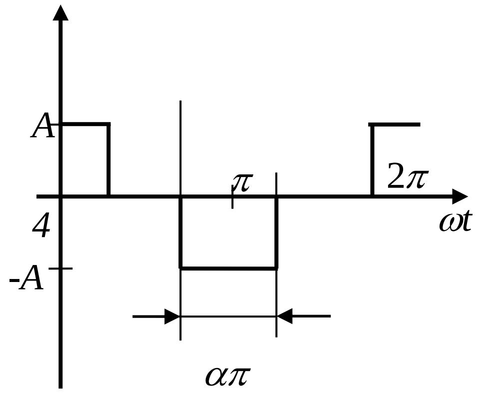
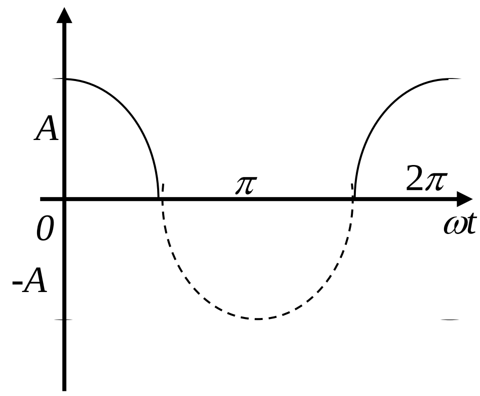
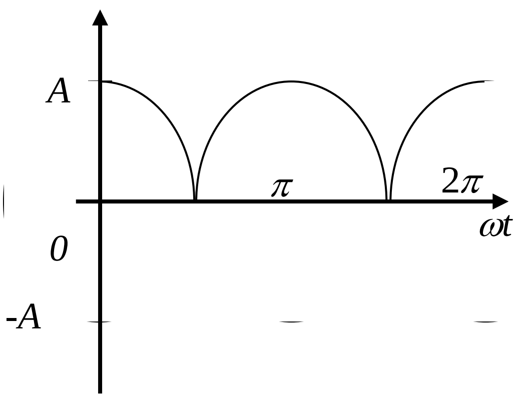
30. Find the constant time τ of transient process if given: R1=8 Ω, R2=3 Ω, R3=6 Ω, L=100 mH.



Apendixes.

Appendix A. Fourier series of Functions with Periodicity 2π

Graphics of Functions	Fourier Series
	$f(\omega t) = \frac{4A}{\alpha\pi}(\sin\alpha \sin\omega t + \frac{1}{9}\sin 3\alpha \sin 3\omega t + \frac{1}{25}\sin 5\alpha \sin 5\omega t + \frac{1}{49}\sin 7\alpha \sin 7\omega t + ...)$
	$f(\omega t) = \frac{8A}{\pi^2}(\sin\omega t - \frac{1}{9}\sin 3\omega t + \frac{1}{25}\sin 5\omega t - \frac{1}{49}\sin 7\omega t + \ldots)$
	$f(\omega t) = \frac{4A}{\pi}(\sin\omega t + \frac{1}{3}\sin 3\omega t + \frac{1}{5}\sin 5\omega t + \frac{1}{7}\sin 7\omega t + ...)$

	$f(\omega t)=\frac{4A}{\pi}(\sin\frac{\alpha\pi}{2}\cos\omega t+\frac{1}{3}\sin\frac{3\alpha\pi}{2}\cos 3\omega t+$ $+\frac{1}{5}\sin\frac{5\alpha\pi}{2}\cos 5\omega t+\frac{1}{7}\sin\frac{7\alpha\pi}{2}\cos 7\omega t+\ldots)$
	$f(\omega t)=\frac{2A}{\pi}(\frac{1}{2}+\frac{\pi}{4}\cos\omega t+\frac{1}{1\cdot 3}\cos 2\omega t-$ $-\frac{1}{3\cdot 5}\cos 4\omega t+\frac{1}{5\cdot 7}\cos 6\omega t-\ldots)$
	$f(\omega t)=\frac{4A}{\pi}(\frac{1}{2}+\frac{1}{1\cdot 3}\cos 2\omega t-\frac{1}{3\cdot 5}\cos 4\omega t+$ $+\frac{1}{5\cdot 7}\cos 6\omega t-\frac{1}{7\cdot 9}\cos 8\omega t-\ldots)$

Appendix B. A short table of Laplace transforms, in each case p is assumed to be sufficiently large that the transform exists.

$f(t)$	$F(p) = \mathbf{L}[f(t)]$	$f(t)$	$F(p) = \mathbf{L}[f(t)]$
1	$1/p$	$\delta(t)$	1
t	$1/p^2$	$\delta(t-c)$	e^{-pc}
t^n	$\dfrac{n!}{p^{n+1}}$	$\delta'(t-c)$	pe^{-pc}
e^{at}	$\dfrac{1}{p-a}$	$u(t-c)$	$\dfrac{1}{p}e^{-pc}$
te^{at}	$\dfrac{1}{(p-a)^2}$	$(t-c)^n u(t-c)$	$\dfrac{n!}{p^{n+1}}e^{-pc}$
$t^n e^{at}$	$\dfrac{n!}{(p-a)^{n+1}}$	$(t-c)^n e^{a(t-c)}u(t-c)$	$\dfrac{n!}{(p-a)^{n+1}}e^{-pc}$
$\sin \omega t$	$\dfrac{\omega}{p^2+\omega^2}$	$\sin \omega(t-c)u(t-c)$	$\dfrac{\omega}{p^2+\omega^2}e^{-pc}$
$\cos \omega t$	$\dfrac{p}{p^2+\omega^2}$	$\cosh a(t-c)u(t-c)$	$\dfrac{p}{p^2-a^2}e^{-pc}$
$\sinh at$	$\dfrac{a}{p^2+a^2}$	$\sin \omega t$ of period π/ω	$\dfrac{\omega}{p^2+\omega^2}\coth\dfrac{p\pi}{2\omega}$
$e^{at}\sin \omega t$	$\dfrac{\omega}{(p-a)^2+\omega^2}$	$\dfrac{1}{t}(e^{at}-e^{bt})$	$\ln\dfrac{p-a}{p-b}$
$e^{at}\cos \omega t$	$\dfrac{p-a}{(p-a)^2+\omega^2}$	$\dfrac{2}{t}(1-\cosh at)$	$\ln\dfrac{p^2-a^2}{p^2}$
$t \sin \omega t$	$\dfrac{2\omega p}{(p^2+\omega^2)^2}$	$\dfrac{2}{t}(1-\cos at)$	$\ln\dfrac{p^2+a^2}{p^2}$
$1 - \cos \omega t$	$\dfrac{\omega^2}{(p^2+\omega^2)p}$	$\dfrac{\sin \omega t}{t}$	$\tan^{-1}\dfrac{\omega}{p}$
ωt - $\sin \omega t$	$\dfrac{\omega^3}{(p^2+\omega^2)p^2}$	$\sin \omega t$- $\omega t \cos \omega t$	$\dfrac{2\omega^3}{(p^2+\omega^2)^2}$
$\sin \omega t + \omega t \cos \omega t$	$\dfrac{2\omega p^2}{(p^2+\omega^2)^2}$	$\cos at - \cos bt$	$\dfrac{(b^2-a^2)p}{(p^2+a^2)(p^2+b^2)}$

BIBLIOGRAPHY

1. Бессонов Л.А. Теоретические основы электротехники. Электрические цепи. – М.: Высшая школа, 1996. – 638 с.

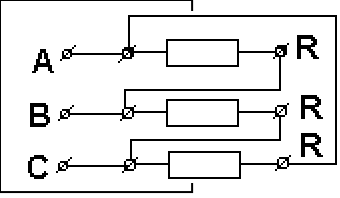
2. Основы теории цепей /Г.В. Зевеке, П.А. Ионкин, А.В. Нетушил, С.В. Страхов. – М.: Энергоатомиздат, 1989.– 528 с.

3. Нейман Л.Р., Демирчян К.С. Теоретические основы электротехники. Т.1 – Л.: Энергоиздат, 1981. – 536 с.

4. Атабеков Г.И. Теоретические основы электротехники. Ч.1. – М.: Энергия, 1978. – 592 с.

5. Шебес М.Р. Теория линейных электрических цепей в упражнениях и задачах. – М.: Высшая школа, 1967. – 478 с.

6. Каплянский А.Е., Лысенко А.П., Полотовский Л.С. Теоретические основы электротехнтки. М.: Высшая школа. –1972. 448 с.



Хілов Віктор Сергійович

МЕТОДИЧНІ ВКАЗІВКИ
І КОНТРОЛЬНІ ЗАВДАННЯ К ПРАКТИЧНИМ ЗАНЯТТЯМ
з дисципліни
“Теоретичні основи електротехніки”

(модулі 3 і 4)

для студентів денної і заочної форм навчання за напрямками підготовки
6.050702 Електромеханіка, 6.050701 Електротехніка та електротехнології

Комп’ютерний дизайн В.С. Хілова

Підписано до друку . . . Формат 30 х 42/4.
Папір офсет. Ризографія. Ум. друк. арк. 4,0.
Обл.-вид. арк. 4,0. Тираж прим. Зам. №

Національний гірничий університет
49005, м. Дніпропетровськ, просп. К. Маркса, 19.